Investigation of Nonlinear Axial Rotor Oscillations of the Multistage Centrifugal Compressor with the Automatic Balancing Device

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- centrifugal compressor, face gap
- pressure difference regulator
- axial force, amplitude frequency characteristic, transient process, dynamic stability

Abstract

In this paper is represented the method of dynamic analysis of rotor vibrations of multistage centrifugal compressor with automatic balancing device based on nonlinear mathematical model that determines the axial movement of the rotor and leakages through hydraulic throttling gaps with non-stationary components. To analyze nonlinear oscillations the method of linearization is used. Dynamic stability of the system was investigated by Hurwitz criterion.

INTRODUCTION

Axial rotor balancing in multistage centrifugal air or gas compressors is carried out mainly by using the unloading plungers. The residual axial force is unloaded by end bearings. Leakage of working gas is limited by end seals. Such systems are the most often applied constructions, but they are complicated. They consist of piston, seal and axial bearing.

Paper [1] presents a new design of the axial forces balancing device of the multistage centrifugal compressor, which has automatic unloading disc working in sealing liquid. That balancing device completely closes the air or gas compressor. Regulator of flow rate of sealing liquid provides right quantum of liquid under the disc. Paper [2] presents the static and flow characteristics of closing automatic rotor balancing device (CARBD) in the multistage centrifugal compressor. These characteristics are obtained basing on equations of rotor axial equilibrium and flow balance. These devices are complex gas-liquid-dynamic or gas- dynamic system with feedback that under certain conditions may have intensive self-oscillations, which affects the vibration state of the compressor.

In papers [3-4] is represented methodology of static and dynamic calculations of the centrifugal machine rotor characteristics.


The results of those articles require additional work which would take into account the turbulent gas flow regime, refinement of expressions for calculation leakages through throttling gaps. New design must include the regulator of pressure difference (RPD) to ensure the constant pressure difference between closing and working air or gas.

MATERIAL AND METHODS (EXPERIMENT)

Scheme of the closing automatic rotor-balancing device is represented in Figure 1.

Figure 1 CARBD scheme: 1 – unloading disc; 2 – cylindrical gap; 3 – chamber on front of disc; 4 – face gap; 5 – pulse channel; 6 – chamber behind of disc; 7 – outlet axial gap; 8 – RPD housing; 9 – membrane; 10, 17 – spring elements; 11 – rod; 12 – valve head; 13 – RPD inlet chamber; 14, 15 – membrane chambers; 16 – last stage impeller
The principle of this device operation is: axial force \( T \) acting to the rotor is unloaded by disk 1. Closing air or gas is supplied to the chamber on front of disc, through the gap 2. Pressure in the chambers 3 and 6 depends on the axial gap \( z \). Random change of axial force \( T \) changes the value of \( z \). Thus, pressure difference \( (p_2 - p_3) \) takes the value that provides equality of unloading force \( F \) and axial force \( T \).

**CARBD of multistage centrifugal compressor** is the automatic control system for which the axial gap \( z \) and closing air or gas leakages \( Q_r \) are controlled variables, axial force \( F \) is regulating action; axial force \( T \), discharge pressure \( p_1 \) and closing air or gas pressure \( p_r \) are external factors.

**MATERIAL AND METHODS (MODEL)**

Dynamic analysis of the CARBD is to determine the dynamic characteristics of the system “rotor-CARB-RPD” based on the equations of axial movement of the compressor rotor and the RPD rod:

\[
m_z \ddot{z} + c_z \dot{z} + k_{spr} z = F - T + F_{spr}; \quad m_x \ddot{x} + c_x \dot{x} + k_{reg} x = F_{reg} - F_m \quad (1)
\]

\[
Q_{cam}'' = V_{cam} \partial_{cam} / E; \quad Q_{s}'' = V_s \partial_s / E; \quad Q_{v}'' = V_v \partial_v / A; \quad Q_{cam}'' = s_x \dot{x}; \quad Q_{s}'' = s_w \dot{w}; \quad Q_{v}'' = Q'' = s_x \dot{x};
\]

\[
V_{cam}, V_s, V_v, V_g \text{ – volumes of hydraulic path chambers; } E = \text{adiabatic modulus of closing air or gas; } s_x = \text{contact area of the RPD saddle.}
\]

![Figure 2 Hydraulic path scheme](http://dx.doi.org/10.12776/mie.v1213-4.210)

Leakage through throttling cylindrical or face gaps are determined by dependences:

\[
Q_{0} = g_x \left( \partial_0 - \partial_{cam} \right); \quad Q_{cam} = g_x \sqrt{\partial_{cam}^2 - \partial_x^2}; \quad Q_{s} = g_x \left( \partial_s - \partial_x \right);
\]

\[
Q_{v} = g_x \partial_v; \quad Q_{reg} = g_x \sqrt{\partial_{reg}^2 - \partial_v^2}; \quad Q_{s} = g_x \partial_s; \quad Q_{v} = Q'' = s_x \dot{x};
\]

where \( g_x, g_v, g_r = g_3 u^{1.5} \) – conductivity of throttling cylindrical and face gaps; \( g_{in}, g_{ex}, g = g_4 u^{1.5} \) – conductivity of RPD throttling gaps; \( u = z/2z ; \xi = x/2z \) – dimensionless gap values; \( g_{in}, g_{ex} \) – base conductivities for nominal values of \( x, z ; p_1 \) – discharge pressure; \( p_m \) – inlet pressure in RPD; \( p_r \) – closing pressure; \( p_{cam}, p_1, p_3 \) – chambers pressure; \( p_4 \) – outlet pressure.

Taking into account the expressions (1) – (4) allows obtaining the system of equations for dynamic analysis:

\[
\begin{align*}
(m_z \ddot{z} + c_z \dot{z} + k_{spr} z) &= s_x (\partial_2 - \partial_1) - \dot{O} + k_{reg}; & m_x \ddot{x} + c_x \dot{x} + k_{reg} x &= k_{reg} \Delta_{reg} - s_w (p_m - p_r); \\
g_x (p_m - \partial_{cam}) &= g_x \partial_0^{0.5} - \partial_{reg} + V_{cam} \partial_{cam} / E + s_x \dot{x} = \\
g_x (\partial_2 - \partial_1) + V_p \dot{p}_2 / E + s_x \dot{x} &= g_x \partial_0^{0.5} - \partial_{reg} + V_p \dot{p}_2 / E + s_x \dot{x} = \\
g_x \partial_0^{0.5} - \partial_{reg} + V_p \dot{p}_2 / E + s_x \dot{x} &= g_x \partial_0^{0.5} - \partial_{reg} + V_p \dot{p}_2 / E + s_x \dot{x} =
\end{align*}
\]

**MATERIAL AND METHODS (CALCULATIONS)**

The system of nonlinear differential equations (4) can not be solved analytically. Further research is conducted for variations of time-variables parameters (“\( \delta \)” is variation sign) by linearization \( z = z_0 + \delta z \), \( u = u_0 + \delta u \), \( x = x_0 + \delta x \), \( \xi = \xi_0 + \delta \xi \), \( T = T_0 + \delta T \), \( F = F_0 + \delta F \), \( p_1 = p_{10} + \delta p_1 \), \( p_m = p_{10} + \delta p_m \), \( p_{cam} = p_{10} + \delta p_{cam} \), \( p_r = p_{20} + \delta p_r \), \( p_3 = p_{30} + \delta p_3 \), \( p_4 = p_{40} + \delta p_4 \) relatively to stationary values (with index “0”) as a result of solving the system of algebraic equations of static analysis.
\[
\begin{align*}
\left[ s_0 (\partial_{20} - \partial_{30}) = T_0 - k_{pp}\Delta; \quad s_m (p_{m0} - p_{10}) = k_{reg}\Delta_{reg}; \\
\Delta_{reg}\right.
\end{align*}
\]
\[
\begin{align*}
\left[ g_m (\partial_{10} - \partial_{20}) = g_0 \sqrt{\Delta_{camb}} - \Delta_{20}^2 = g_1 (\partial_{20} - \partial_{10}) + g_2 \partial_{20} \sqrt{\Delta_{20}^2 - p_{20}^2}; \\
g_0 \sqrt{\Delta_{camb}^2 - p_{20}^2} = g_1 \sqrt{\partial_{20}^2 - p_{20}^2} + g_2 \partial_{20} \sqrt{\partial_{20}^2 - p_{20}^2};
\end{align*}
\]
relatively to parameters \( u_{0}, \dot{e}, p_{camb}, p_{m0}, p_{10}, p_{20} \).

For further calculations are used dimensionless parameters: \( \delta \hat{\partial}_{camb} = \hat{\delta}_{camb}/\partial_{camb}; \quad \delta \hat{e} = \delta e/\partial_{camb}; \quad \delta \psi_1 = \delta \psi_1/\partial_{camb}; \quad \delta \psi_2 = \delta \psi_2/\partial_{camb}; \quad \delta \psi_3 = \delta \psi_3/\partial_{camb}; \quad \delta T = 5T/(p_{10}); \quad \delta p = 5(p_0), \) where \( p_0 \) is base pressure value corresponding to the nominal discharge pressure \( p_{10} \) of the compressor; \( s_0 = T_0/p_0 \) – base area as the ratio of nominal axial force \( T_0 \) to pressure \( p_0 \).

In parameters \( \sigma = b \delta \psi_1, \delta p = 5(p_0), (b \ – \ proportionality \ coefficient, \sigma = s_0/s_0 – \ dimensionless \ effective \ area) \) system of dynamics equations (4) can be represented in matrix and operator form

\[
N(p)\partial U = B\partial \psi',
\]

where

\[
N(p) = \begin{bmatrix}
K_1(T^2 + 2\xi T + 1) & 0 & 0 & 0 & -\sigma & \sigma \\
0 & K_1(T^2 + 2\xi T + 1) & 0 & \sigma_1 & 0 & 0 \\
0 & 0 & K_1(T^2 + 1) & -K_1 & 0 & 0 \\
-K_1 & 0 & 0 & T_{p+1} + K_1 & T_{p+1} + -K_1 & 0 \\
K_1(T_p + 1) & T_p & -K_1 & T_{p+1} & T_{p+1} & K_1 \\
-K_{12} & 0 & 0 & 0 & -K_{12}(\tau + 1) & T_{p+1}
\end{bmatrix}
\]
is matrix of differentiation operators; \( \partial U = (\partial u, \partial e, \partial \psi_1, \partial \psi_2, \partial \psi_3, \partial \phi, \partial \theta)^T \) is reaction of the system to the external action \( B\partial \psi', \) where \( B = [-b \sigma, 0, 0, K_1, 0]^T. \)

The matrix \( N(p) \) contains 28 constant parameters: time constants \( T_{1,2,3,6,7} \) damping coefficients \( \zeta_{1,2} \) and amplification factors \( K_{1,2,3,5,13}. \)
The matrix $N(\omega)$ can be decomposed into real and imaginary parts ($i$ is imaginary unit, $\omega$ is angular frequency of the rotor):

$$N(\omega) = N_{Re}(\omega) + i\omega N_{Im}(\omega).$$

Real $\mathbf{U}$ and imaginary $\mathbf{V}$ parts of the vector of frequency transfer functions $W(\omega) = [N(\rho)]^{-1}B = \mathbf{U}(\omega) + i\omega\mathbf{V}(\omega)$ are

$$\mathbf{U} = \left(N_{Re}N_{Im}N_{Re} + \omega^2 N_{Im}\right)^{-1}N_{Re}N_{Im}^{-1}B; \quad \mathbf{V} = -\left(N_{Re}N_{Im}N_{Re} + \omega^2 N_{Im}\right)^{-1}B.$$  

Vector of amplitude and phase frequency characteristics consists of modules and phases of elements of vector of $W(\omega)$:

$$A_i(\omega) = \sqrt{\mathbf{U}^T(\omega)+\mathbf{V}^T(\omega)}; \quad \phi_i(\omega) = \arctg \frac{\omega \mathbf{V}(\omega)}{\mathbf{U}(\omega)} \quad (i = u, \xi, \text{cam, } \dot{a}, 2, 3).$$

To ensure dynamic stability of the system it is necessary to all real roots of characteristic equation $|N(\rho)| = a_0\lambda^8 + a_1\lambda^7 + \ldots + a_8$ were negative. By Hurwitz criterion this condition is satisfied if coefficients $a_{0,8}$ and the main diagonal minors of the matrix $\Delta$ are positive:

$$\Delta = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & 0 & 0 & 0 & 0 \\
a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & 0 & 0 & 0 \\
a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & 0 & 0 & 0 \\
a_4 & a_5 & a_6 & a_7 & a_8 & 0 & 0 & 0 & 0 \\
a_5 & a_6 & a_7 & a_8 & 0 & 0 & 0 & 0 & 0 \\
a_6 & a_7 & a_8 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_7 & a_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 \end{bmatrix}.$$  

### RESULTS AND DISCUSSION

Numerical calculations are carried out for the compressor K 180-131-1. Initial data for dynamic analysis are parameters of the static calculations: $p_0 = 4.6$ MPa, $s_b = 0.039$ m$^2$, $z_o = 0.15$ mm; $u_0 = 0.88$, $\xi_0 = 0.59$; $s_x = 3.9 \times 10^{-5}$ m$^2$; $s_m = 3.9 \times 10^{-8}$ m$^2$, $s_x = 0.08$ m$^2$; $p_{00} = 5.1$ MPa, $p_{20} = 4.8$ MPa, $p_{30} = 2.6$ MPa, $p_4 = 0$; $g_{m} = 2.8 \times 10^{-5}$ m$^2$/Pa-s; $g_{a} = 2.3 \times 10^{-9}$ m$^2$/Pa-s; $g_{b} = 2.7 \times 10^{-9}$ m$^2$/Pa-s. $g_{c} = 2.3 \times 10^{-9}$ m$^2$/Pa-s; $g_{d} = 3.7 \times 10^{-10}$ m$^2$/Pa-s; $g_{1} = 4.7 \times 10^{-10}$ m$^2$/Pa-s and following parameters: $m_2 = 350$ kg, $m_3 = 0.65$ kg, $E = 1.42 \times 10^5$ Pa; dynamic viscosity of closing gas $\mu = 1.82 \times 10^{-3}$ Pa-s; damping coefficients $c_i = 0.1\mu(i\lambda_0^2 - \lambda_0^4)/\gamma_0 = = 6.4 \times 10^{-4}$ N/m/s; $c_s = c_c = 6.4 \times 10^{-4}$ N/s/m; parameters (8) are $T_1 = 5.4$ ms, $T_2 = 1.2$ ms, $T_3 = 8.7$ ms, $T_4 = 1.8 \times 10^{-2}$ ms, $T_5 = 5.4 \times 10^{-2}$ ms, $T_6 = 1.6 \times 10^{-2}$ ms, $T_7 = 4.8 \times 10^{-4}$ ms, $T_8 = 5.8 \times 10^{-3}$ ms, $T_9 = 7.5 \times 10^{-3}$ ms, $T_{10} = 0.02$ ms, $T_{11} = 5.4 \times 10^{-4}$ ms, $T_{12} = 4.9 \times 10^{-4}$ ms, $T_{13} = 6.9 \times 10^{-4}$ ms; $\zeta_0 = 0.05$, $\xi_0 = 0.05$; $\xi_1 = 0.22$; $K_1 = 0.01$, $K_2 = 3.5 \times 10^{-8}$, $K_3 = 0.02$, $K_4 = 0.25$, $K_5 = 0.02$, $K_6 = 0.15$, $K_7 = 0.99$, $K_8 = 0.09$, $K_9 = 0.87$, $K_{10} = 0.04$, $K_{11} = 0.04$, $K_{12} = 0.68$, $K_{13} = 0.55$; $\delta\phi_{1a} = 0.2$. Angular velocity of shaft rotation $\omega_0 = 1480$ rad/s. Amplitude frequency characteristic is represented on Figure 3.

**Figure 3** Amplitude frequency characteristic for axial oscillations of the rotor

The first and second resonance frequencies of axial oscillations of the rotor are $\omega_1 = 2190$ rad/s and $\omega_0 = 5360$ rad/s. Operating frequency $\omega_0 = 0.7\omega_1$ and $\omega_0 = 2.4\omega_1$. Resonance amplitudes of axial oscillations of the rotor are $A_1 = 62$ mm, $A_3 = 60$ mm. Amplitude on the operating mode $A_0 = 3$ mm corresponds to accident-free mode.

Positive values of characteristic equation coefficients

$\alpha_0 = 2.65 \times 10^{-10}$; $\alpha_1 = 8.8 \times 10^{-34}$; $\alpha_2 = 8.4 \times 10^{-28}$; $\alpha_3 = 2.2 \times 10^{-22}$; $\alpha_4 = 1.6 \times 10^{-17}$; $\alpha_5 = 1.0 \times 10^{-14}$; $\alpha_6 = 5.4 \times 10^{-10}$; $\alpha_7 = 5.0 \times 10^{-8}$; $\alpha_8 = 2.2 \times 10^{-3}$ and main diagonal minors of matrix (12) indicate stability of the dynamic system. In addition, numerical roots of characteristic equation ($\lambda_1 = -1.8 \times 10^6$, $\lambda_2 = -1.1 \times 10^5$, $\lambda_3 = -2.2 \times 10^5$, $\lambda_4 = -1.3 \times 10^6$, $\lambda_5 = -6.0 \pm 2.19i$, $\lambda_6 = -93.1 \pm 5.36 \times 10^4$) have negative real parts. Modules of imaginary parts of roots $\lambda_5, \lambda_6$ are equal to natural frequencies of the system: $\omega_1 = 2190$ rad/s, $\omega_0 = 5360$ rad/s.

Figure 4 represents transient characteristics of CARBD. Control time is $t_0 = 40$ ms, maximum over-control for the rotor is 11 mm.

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NEW APPROACHES FOR PROBLEM SOLUTION

The mathematical model, which is stated in this article, is a system of nonlinear differential equations of 8th order that describes rotor and rod motions and leakages through laminar and turbulent cylindrical and face throttling gaps. Described method of dynamic analysis of the dynamic system “rotor – automatic axial balancing device – regulator of pressure difference” – is based on this model.

Research provides an opportunity to build amplitude and phase frequency characteristics, calculate natural frequencies of axial oscillations of the rotor and check dynamic stability of the system. The results can be used for design calculations of rotor vibration state of the multistage centrifugal compressor with automatic balancing device.

CONCLUSIONS AND FUTURE DIRECTION OF RESEARCH

The closing automatic rotor-balancing device of the multistage centrifugal compressor with regulator of pressure difference acts as the end seal and the hydrostatic bearing with self-regulating gap and gas leakages. The main advantages of this design are absence of end seals and bearings and leakage of working gas.

Amplitude frequency characteristic of rotor axial oscillations and transient processes and dynamic stability of the system are represented as example for compressor K 180-131-1

At this stage dynamic stability of the system is implemented by numerical verification by Hurwitz criterion. In perspective it is necessary to define conditions and boundaries for dynamic stability analytically and investigate nonlinear mathematical model without linearization on the basis of Runge-Kutta and Bulirsch-Stoer methods.

REFERENCES