AN UPPER BOUND ANALYSIS OF T-SHAPED EQUAL CHANNEL ANGULAR PRESSING

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Abstract
T-shaped equal channel angular pressing (T-shaped ECAP) is a novel process of severe plastic deformation (SPD). The maximum effective strain per pass in the T-shaped ECAP is larger than that in the ECAP. Deformation models of the T-shaped ECAP have been proposed by a designed grids deformation experiment considering the deformation dead zone (DDZ) formation or without it. Based on the deformation models, an upper bound approach has been used to analyse the T-shaped ECAP process. The effects of the plastic deformation zone (PDZ) angles and the friction factor on the relative pressing pressure and the DDZ are analysed. It is found that the relative pressing pressure increases with an increase of the angle β and the friction factor m, however, the DDZ decreases. The upper bound solution for the T-shaped ECAP with the DDZ is more close to the experimental load than that without the DDZ. The results show that there is a good agreement between the theoretical value and the measured maximum load required for the T-shaped ECAP conducting.

Keywords: T-shaped equal channel angular pressing, upper-bound analysis, grids deformation, severe plastic deformation

1 Introduction
Equal channel angular pressing (ECAP) is an effective route to refining coarse grains into ultrafine-grained or nanostructured states with the grain size of 0.1~1μm by means of large plastic shear deformation [1]. The most advantage of ECAP is that the billet undergoes severe plastic deformation (SPD) with the maximum strain arriving at 1.15 after one single pass but the same cross-sectional geometry of the billet remains. Therefore, it is possible to repeat the pressings for a number of passes in order to achieve a high cumulative strain.

In order to optimize the ECAP process and obtain homogeneous nanostructured billets, many attempts have been made to analyse the ECAP process using deformation mechanics and finite element simulations [2-6]. Segal proposed three types of plastic deformation zone (PDZ) [2, 3]. The pressing pressure, the strain and the shear plane were investigated by the slip line theory. Due to the importance of the PDZ in optimization of the ECAP tool designs and processing variables, four different shapes, namely, i) the single shear plane, ii) the shearing fan, iii) the partial split PDZ mode, and IV) the full split PDZ mode, were assumed and compared to experimental results by Stoica et al [7]. Effects of the PDZ shape on the material flow during the ECAP were analysed by the FEM and the grid deformation patterns [8-13].
Using the upper-bound analysis method, the PDZ shape can be predicted by optimizing the total power loss along all discontinuity and friction surfaces. Lee estimated the pressing pressure and the strain considering the single shear plane of the PDZ [14]. Taking into account the shearing fan with or without the deformation dead zone (DDZ), the upper-bound analysis was conducted [15-17]. Based on the rotational and linear velocity fields, the PDZ is divided into three rotational velocity fields, and then the force needed for ECAP was predicted [18]. Luis and Luri presented an upper-bound analysis of the required forces in the ECAP process considering a three-dimensional geometry [19]. There is a good agreement between the upper-bound solution and experimental forces.

To obtain more severe mechanical working, T-shaped equal channel angular pressing (T-shaped ECAP) was developed by Rao et al. [20, 21]. Assuming that any part of the material will face a gradual change in its velocity in PDZ during the T-shaped ECAP, the strain and the extrusion pressure were calculated by the upper-bound analysis [22]. Although the upper-bound pressure showed a good agreement with the measured extrusion pressure, the proposed analytical deformation pattern was not verified by the experimental grids flow.

In the present work, rigid plastic deformation models incorporating the DDZ based on the grids deformation pattern have been proposed. The upper bound analysis of T-shaped ECAP has been carried out based on the developed models. The predicted pressing pressure from the upper-bound analysis has been compared with the experimental loads.

2 Materials and methods

In the T-shaped ECAP process, the maximum effective strain ~1.39 per pass can be attained in comparison to 1.15 during the ECAP [22]. As shown in Fig. 1(a) during the T-shaped ECAP the billet is pressed down from the vertical channel to two laterally horizontal channels with the same inlet and the exit cross-sections. The biggest difference of the T-shaped ECAP in respect to the ECAP is that in the former case the velocity of the billet in the horizontal channel is half of that in the vertical channel, \( v_1 = 2v_2 = v_0 \). In the upper-bound method, it is assumed that there are a number of velocity discontinuity surfaces, on which the velocity suddenly changes [23]. Consequently, the deformation model I without the DDZ is as shown in Fig. 1(a), in which it contains four rigid plastic bodies.

Talebanpourt et al. proposed a continuous and gradual velocity field, in which the material flows along a specific streamline, which is the elliptical curve [22]. However, the deformation model was not examined by the experiments.

Pure Al (99.7 mass%) billets with a size of 12 × 12 × 80 mm³ were annealed in vacuum at 673K for 1h. The yield stress, \( \sigma_0 \), has been 80 MPa. The shear yield stress, \( k \), has been 46 MPa, where \( k = \sigma_0 / 3 \). Two half-billets with size of 12×6×80 mm³ have been cut, and one has been engraved with grids of 2×2 mm², another has been left free of grids. Then the two half-billets have been glued together and processed by the T-shaped ECAP. Fig. 1(b) shows the grids deformation pattern, in which it is evident to observe the DDZ formation at the bottom edge of the billet. Therefore, in order to describe this experimental observation it is reasonable to propose the deformation model presented in Fig. 1(c). The surfaces of the DDZ are simplified from the arc curves to straight lines because the DDZ is very small.

T-shaped ECAP has been conducted using a hydraulic press of 100 T capacity at room temperature. The billets were pressed one pass using a molybdenum disulfide lubricant. The pressing speed is 1 mm s⁻¹. The load-displacement curve has been recorded during the T-shaped ECAP to compare with the upper bound solutions.
Fig. 1  Deformation models: (a) the model I without the DDZ, (b) the experimental grids deformation, and (c) the model II with the DDZ

3  Upper bound analysis of T-shaped ECAP

On the basis of an extremum principle, the upper-bound solution is equal to or higher than the actually required force in metal forming process. The upper bound on power is defined as:

\[ J' \leq 2k \int \frac{1}{2} \varepsilon_{ij} \varepsilon_{ij} dV + s_f k \Delta \mathbf{v} \mathbf{d}s + s_f mk \Delta \mathbf{v}_f \mathbf{d}s - s_i T_i \mathbf{v}_i \mathbf{d}s \]  

(1.)

where \( J' \) is the actual power that must be supplied to perform the plastic deformation, \( k \) is the shear yield stress, \( \varepsilon_{ij} \) is the strain rate tensor, \( V \) is the volume of the deformation zone, \( S_f \) and \( S_f \) are the area of velocity discontinuity and frictional surfaces, correspondingly, \( |\Delta \mathbf{v}| \) and \( |\Delta \mathbf{v}_f| \) are the relative velocity on \( S_f \) and \( S_f \), correspondingly, \( m \) is the friction factor, \( S_i \) is the area where the tractions may occur, \( T_i \) is the external traction force, \( \mathbf{v}_i \) is the velocity of the surfaces under the force \( T_i \).

The first term in right side of Eq. (1) is the internal power consumed in the PDZ. It is assumed that the PDZ is rigid. So, the first term can be neglected. In the T-shaped ECAP process there is no external tension. So, the last term is equal to zero.
3.1 The deformation model I (without the DDZ)

In Fig. 1(a), BD and BF are velocity discontinuity planes. The velocity hodograph is as shown in Fig. 2. \( v_{BD} \) and \( v_{BF} \) are the relative velocities along the plane BD and BF, correspondingly. \( v_{BC} \) and \( v_{BE} \) are the relative velocities of the material against the die walls, correspondingly. The power for the deformation of material of \( w \) in width \( W \), is given by

\[
W = W_i + W_{SBDF} + W_{SBCD} + W_{SBEF} + W_{swalls}
\]  

Here, \( W_i \) is the power dissipated on the velocity discontinuity planes.

\[
W_i = k (S_{BC} \cdot v_{BC} + S_{BD} \cdot v_{BD} + S_{BF} \cdot v_{BF} + S_{BE} \cdot v_{BE}) = 3k w^2 v_0
\]  

\( W_{SBDF} \), \( W_{SBCD} \) and \( W_{SBEF} \) are the power dissipated on the friction between the PDZ and the die front and back walls, correspondingly.

\[
W_{SBDF} = mkS_{BDF} \cdot v_{AB} = mk w^2 v_0
\]  

\[
W_{SBCD} = mkS_{BCD} \cdot v_{BC} = \frac{1}{4} mk w^2 v_0
\]  

\[
W_{SBEF} = mk(S_{BE} \cdot v_{BE} + S_{BF} \cdot v_{BE}) = \frac{1}{4} mk w^2 v_0
\]  

\[
W_{swalls} = mk v_1 \cdot 4w(L - a) + mk v_2 \cdot 8wb = 4mk w(L - a + b)v_0
\]

where \( a \) is the stroke of the punch, \( b \) is the length of the material which passes the channel intersection region. \( b = \frac{1}{2} a \) due to the volume constancy. When the friction outside of the PDZ is neglected, substituting Eq. (3)-(6) into Eq. (2), the power \( W \) can be expressed as

\[
W = (3 + 1.5m)kw^2 v_0
\]

The external power, \( J \), is given by \( J = qw^2 v_0 \), where \( q \) is the average pressing force per unit area. Therefore the following relation is obtained.
3.2 The deformation model II (with the DDZ)

In Fig. 1(c), BD, BF, BC and BE are velocity discontinuity planes. The velocity hodograph is as shown in Fig. 2(b). \( v_{BD}, v_{BF}, v_{BC} \) and \( v_{BE} \) are the relative velocities along the plane BD, BF, BC and BE, correspondingly. The power for the deformation of material of \( w \) in width, \( W \), is given by

\[
W = W_i + W_{S_{BDF}} + W_{S_{BCD}} + W_{S_{BEF}} + W_{S_{walls}} \tag{10}
\]

Here, \( W_i \) is the power dissipated on the velocity discontinuity planes.

\[
W_i = k \cdot S_{BC} \cdot v_{BC} + S_{BD} \cdot v_{BD} + S_{BF} \cdot v_{BF} + S_{BE} \cdot v_{BE}
\]

\[
= 2k \left( \frac{w^2}{2 \sin \beta} \cdot \frac{v_0 \sin \alpha}{\sin \pi - \alpha - \beta} + \frac{w^2}{2 \sin \alpha} \cdot \frac{v_0 \sin \beta}{\sin \pi - \alpha - \beta} \right)
= \frac{kw^2v_0}{\sin \alpha + \beta} \cdot \frac{\sin \alpha + \sin \beta}{\sin \alpha} \tag{11}
\]

\( W_{S_{BDF}}, W_{S_{BCD}} \) and \( W_{S_{BEF}} \) are the powers dissipated on the friction between the PDZ and the die front and back walls, correspondingly.

\[
W_{S_{BDF}} = mkS_{BDF} \cdot v_{AB} = 2mk \cdot \frac{1}{2}w \cdot \frac{w}{2 \tan \alpha} \cdot v_0 = \frac{mkw^2v_0}{2 \tan \alpha} \tag{12}
\]

\[
W_{S_{BCD}} = mkS_{BCD} \cdot v_{BC} = 2mk \cdot \frac{1}{2}w \cdot \frac{w}{2 \sin \beta} \cdot v_0 \sin \alpha = \frac{mkw^2v_0 \sin \alpha}{2 \sin (\alpha + \beta)} \tag{13}
\]

\[
W_{S_{BEF}} = mkS_{BEF} \cdot v_{BE} = \frac{mkw^2v_0 \sin \alpha}{2 \sin (\alpha + \beta)} \tag{14}
\]

When the friction outside of the PDZ is neglected, \( W_{S_{walls}} \) is equal to zero. Substituting Eq. (11) ~ (14) into Eq. (10), the power \( W \) can be expressed as

\[
W = \frac{kw^2v_0}{\sin \alpha + \beta} \cdot \frac{\sin \alpha + \sin \beta}{\sin \alpha} + mkw^2v_0 \left( \frac{1}{2 \tan \alpha} + \frac{\sin \alpha}{\sin (\alpha + \beta)} \right) \tag{15}
\]

The external power, \( J \), is given by \( J = qw^2v_0 \), where \( q \) is the average pressing force per unit area. Therefore the following relation is obtained.

\[
q \leq \frac{k}{\sin \alpha + \beta} \cdot \frac{\sin \alpha + \sin \beta}{\sin \alpha} + mk \left( \frac{1}{2 \tan \alpha} + \frac{\sin \alpha}{\sin (\alpha + \beta)} \right) \tag{16}
\]

When \( \beta = 90^\circ \) and \( \alpha = 26.565^\circ \), the DDZ disappears. Thus the deformation model II is identical to the model I, and then the Eq. (16) is reduced to the Eq. (9).

4 Results and discussion

From Fig. 1(c) and Eq. (16), it follows that the development of the DDZ and the pressing force are affected by the angles, \( \alpha \) and \( \beta \) and the friction factor \( m \). Taking the midpoint of the segment CD as the circle center and the half of the segment CD as the radius, the line EC and the circle are
tangent to the point C, and the line AB tangent to the point B. At this moment, the largest DDZ comes into being, and \( \alpha = \beta = 45^\circ \) as shown in Fig. 3(a). However, it is not easy for the billet itself to develop the largest DDZ.

![Fig. 3](image)

*Fig. 3* Effect of the angle \( \beta \) on the DDZ (a) \( \beta = 45^\circ \) and (b) \( \beta = 75^\circ \)

When BD = CD, namely, the point B and C are on the same circle taking the point D as the center, the line EC and the line AB, tangent to the circle at the point C, intersect at the point B, respectively. In a similar way, the length EF = BF taking the point F as the center of a circle. In this case, a billet could flow in a gradual and continuous way from the vertical channel to the horizontal channels, and \( \alpha = 30^\circ \) and \( \beta = 75^\circ \) as shown in Fig. 3(b). Thus, the angle \( \beta \) is ranged from \( 45^\circ \) to \( 90^\circ \).

![Fig. 4](image)

*Fig. 4* Effect of (a) the angle \( \beta \) and (b) the constant friction factor \( m \) on the relative pressure calculated by Eq. (16)

*Fig. 4* shows the effect of the angle \( \beta \) and the friction factor \( m \) on the relative pressure. In *Fig. 4*(a), the relative pressure increases with an increase of the angle \( \beta \), correspondingly, the DDZ decreases. For the ideally plastic material, the friction coefficient \( m \) varies between 0 and 1. The constant friction factor \( m \) has a significant effect on the relative pressure as shown in *Fig. 4*(b). The friction force along all of the surfaces between the material and the die is modelled by \( mk \). For commercial pure aluminum, the constant friction factor \( m \) is assumed as 0.1 [18, 24]. Taking into account the existence of the friction force along all of the surfaces between the material and the die, the upper-bound solution to the T-shaped ECAP has been compared to the experimental load-displacement curve for the first pass in *Fig. 5*. Without consideration of the DDZ, the upper bound pressure predicted by Eq. (9) is 36.4 KN, 28% higher than the maximum experimental
load for the T-shaped ECAP. In case of $\beta = 75^\circ$ with the DDZ, the upper bound pressure predicted by Eq. (16) is 33.1 KN, 16% higher than the maximum experimental load. In fact, $\beta$ is close to $65^\circ$ from the grids deformation pattern in Fig. 1(b). The predicted pressure is 31.3 KN, which is very close to the maximum experimental load 28.5 KN, the difference between them is only 10%, showing a good agreement between the theoretical model and experiments.

Fig. 5 A typical load-displacement curve for the first pass in the T-shaped ECAP process in comparison to the upper bound solutions

Furthermore, the angle $\beta$ measured from the grids deformation in Fig. 1(b) is $65^\circ$, less than the theoretical value $75^\circ$. The DDZ decreases with an increase of the angle $\beta$ in Fig. 3. It indicates that the DDZ increases with increasing the friction factor. It is well in agreement with that in the ECAP process, in which the PDZ becomes larger with the friction factor due to the DDZ increase [12, 15, 18, 25].

5 Conclusions
An upper bound approach has been used to analyse the T-shaped ECAP process based on the proposed deformation models. The effects of the PDZ angles and the friction factor on the relative pressing pressure and the DDZ have been analysed. The following conclusions can be outlined:

1. Deformation models taking into account the existence or absence of deformation dead zone (DDZ) in the T-shaped ECAP process are proposed based on the results of a designed grids deformation experiment.
2. The DDZ and the pressing force in the T-shaped ECAP are affected by the angles $\alpha$, $\beta$ and the friction factor $m$.
3. The relative pressing pressure increases with an increase of the angle $\beta$ and the friction factor $m$, however, the DDZ decreases.
4. The upper bound solution to the T-shaped ECAP with DDZ is more close to the experimental load than that without DDZ. From the grids deformation pattern it follows that the angle $\beta$ is close to $65^\circ$. In such a case, the difference between the predicted pressure and the maximum experimental load is only 10%, showing a good agreement
between the theoretical value and the measured maximum load required for T-shaped ECAP.

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