A LUMP-INTEGRAL MODEL FOR FREEZING AND MELTING OF A BATH MATERIAL ONTO A PLATE SHAPED SOLID ADDITIVE IN AN AGITATED BATH

Umesh Chandra Singh\textsuperscript{1)}, Anant Prasad\textsuperscript{2),} Arbind Kumar\textsuperscript{3})

\textsuperscript{1) Engineering Division, Tata Steel Limited, Jamshedpur, India}
\textsuperscript{2) Retd, Department of Mechanical Engineering, National Institute of Technology, Jamshedpur, India}
\textsuperscript{3) Department of Mechanical Engineering, Birla institute of Technology Mesra, Ranchi, India}

Received: 23.01.2012
Accepted: 26.11.2012

Corresponding author: e-mail: umeshchandra.singh@tatasteel.com, Tel: 091 0657 6648083, Engineering Division, Tata Steel Limited, Business Centre, Jamshedpur-831001, India.

Abstract

A lump integral model is developed for freezing and melting of the bath material onto the surface of a plate shaped additive immersed in an agitated melt bath. It exhibits the dependence of this occurrence on independent parameters-the initial temperature, $\theta_{ai}$ of the additive, the bath temperature, $\theta_{b}$, the Biot number, $B_i$ the property ratio, $B$ and the Stefan number, $S_t$ and yields closed-form solutions for time variant frozen layer thickness, $\xi$ around the additive and heat penetration depth, $\eta$ in the additive. In the solutions, $B$, $B_i$, $\theta_{b}$ and $\theta_{ai}$ appear as a conduction factor, $C$ of that ranges from 0 to $\infty$. The frozen layer thickness per unit $S_t$ with respect to $C$ takes time $\tau_{cmax}=1/3$ for its maximum growth whereas this maximum thickness $\xi_{cmax}$ becomes $(1- \theta_{ai})/3$. The total time of the growth of the maximum frozen layer thickness with its subsequent melting, $\tau_{ct}$ is 4/3 when the heat penetration depth reaches the central axis of the plate additive, $\eta=1$. When $C_{st}\rightarrow0$ signifying highly agitated bath ($h\rightarrow\infty$) or additive preheated to the freezing temperature of the bath material, no freezing of the bath material occurs. For the bath at the freezing temperature of the bath material, the frozen thickness is also obtained. The model is validated by reducing the present problem to heating of the plate additive subjected to a constant temperature maintained at the freezing temperature of the bath material.

Keywords: Mathematical modeling; melt-additive system; freezing, melting.

1 Introduction

Melting of a solid additive in a melt bath is employed in manufacture of steel, alloy, cast iron and similar other materials. It undergoes different steps. The first step is freezing and melting of the bath material around the surface of the additive along with rise in the temperature of the additive. In the second step, the additive is heated to its melting temperature after its emergence at an elevated temperature whereas it melts in the third step. These steps depend on the temperatures of the bath and the additive, bath agitation and thermo-physical properties of the additive- bath system and take certain time for their completion. Such a time regulates the productivity of manufacture of these materials. Since their increased productivity without compromise of quality for global competitiveness is of great importance, the reduction in the time of the production is essential. It can be achieved, once the time taken in the first step, which is not needed in the melting process of the additive but occurs due to requirement of heat to be
conducted in the additive more than the convective heat available from the bath during its initial period resulting in supply of latent heat of fusion by freezing of the bath material onto the additive and at its later period less than this available convective heat causing the excess of the convective heat to melt the frozen layer, is minimized. This is possible with the growth of a smaller thickness of the frozen layer. It is attained when the convective heat from the bath is increased by increasing the bath agitation. It essentially reduces the frozen layer thickness, the time of completion of the first step and does not allow the heat to penetrate the entire volume of the additive. Consequently, the total time of melting is decreased and the productivity of manufacturing is increased.

Investigation of such a situation that leads to thermal resistance of the frozen layer negligible with respect to that of the bath seldom appears in the literature. However, the occurrence of the first step for plate [1], cylindrical [2-4] and spherical [5, 6] shaped solid additives is analyzed when the frozen layer formed on these additives has their thermal resistances comparable with those of their bath. In this situation, it is observed that the increased heat transfer coefficient of the bath reduces the frozen layer thickness and time taken in the first step for the plate [1], spherical [6] and cylindrical [3] additives. This prediction is implicit in [2, 5] whereas in [4] only instant equilibrium temperature at the interface between the additive and the bath immediately after the immersion of the additive in the bath is found. Closed-form solutions for the growth of the maximum frozen layer thickness, its time of development and the total time of freezing and melting of the bath material onto the cylindrical additive in an agitated bath [7] is also reported recently.

In view of these facts, this work aims at development of a lump-integral model in dimensionless form for freezing and subsequent melting of the bath material onto the surface of the plate additive immersed in the agitated bath. The frozen layer formed is assumed to have much smaller thermal resistance than that of the additive. The model exhibits the dependence of this phenomenon on the independent parameters- initial temperature of the additive, $\theta_{ai}$, the Stefan number, $S_t$ indicative of phase change of the bath material, the Biot number, $B_i$ representing the bath agitation, the property ratio, $B$ and the bath temperature, $\theta_b$. It provides closed-form solutions for frozen layer with subsequent melting, its completion time and heat penetration depth within the additive. They are functions of these independent parameters with $B$, $B_i$, $\theta_b$ and $\theta_{ai}$ occurring as a conduction factor, $C_{of}$. Further transformation makes these solutions dependent only on time. For this situation, time for maximum growth of frozen layer and total time for freezing of this layer with its subsequent melting are derived. To validate the model it is converted to a solution of the past investigation. A close agreement is exhibited.

2 Mathematical Model

To estimate the time for the first step as described in the introduction, a suitable mathematical model is designed. Here, the additive is in the form of a plate of thickness $2b$ and at a uniform temperature $T_{ai}$ lower than its melting temperature, $T_{af}$ before its immersion in the agitated melt bath contained in a ladle in which the bath is in a highly agitated state owing to falling stream of the melt during tapping or stirring in the ladle. This bath is assumed to remain at a constant temperature, $T_b$ during the first step. Moreover, the plate additive has the melting temperature, $T_{af}$ higher than the freezing temperature, $T_{mf}$ of the bath material. For such a solid additive-melt bath system exhibited in Fig.1, $T_{ai}<T_{mf}<T_{af}<T_b$ and upon immersion of the plate additive, the melt of the bath immediately freezes onto the surface of the plate, the contact interface between
the frozen layer and the plate arrives at an equilibrium temperature, $T_e$ less than the additive melting temperature, $T_{af}$ and the temperature gradient sets up onto the plate and the frozen layer sides. Due to passing of the immersion time, $T_e$ builds up, the frozen layer grows in thickness, the heat penetrates the additive. This happens when the rate of heat conduction to the plate due to the temperature gradient developed on its side remains more than the convective heat available from the bath. The deficient amount of the convective heat is met by the latent heat of fusion evolved due to freezing of the bath material onto the surface of the plate. Once the two rates of heat transfer become the same, the frozen layer growth ceases. After this time the convective heat available from the bath to the frozen layer is greater than the conductive heat transfer from the frozen layer to the plate. This renders the frozen layer to melt with further rise of the interface temperature and heating of the plate. Ultimately, the complete melting of the frozen layer occurs leaving the plate at an elevated temperature which is less than $T_{af}$. Heat transfer in this situation is regulated by one-dimensional conjugated transient heat conduction. Its non-dimensional integral form describing the temperature field within the heat penetration depth of the plate along with initial and boundary conditions can be written as

$$\frac{d}{d\tau} \int_{0}^{\xi} \theta_a d\xi_a - \theta_a \bigg|_{\xi=0} \frac{d0}{d\tau} + \theta_a \bigg|_{\xi=0} \frac{d\eta}{d\tau} = \frac{1}{B} \left[ \frac{\partial \theta_a}{\partial \xi_a} \bigg|_{\xi=0} - \frac{\partial \theta_a}{\partial \xi_a} \bigg|_{\xi=-\eta} \right]$$

(1.)

$$\theta_a = \theta_{ai} \cdot -\eta < \xi_a < 0, \eta = 0, \tau = 0$$

(2.)

$$\theta_a = \theta_e \leq \theta_{af} \cdot \xi_a = 0, \tau > 0$$

(3.)

$$\partial \theta_a / \partial \xi_a = 0, \quad \theta_a = \theta_{ai} \cdot \xi_a = -\eta, \tau > 0$$

(4.)

Fig. 1  Schematic of freezing and melting of bath material onto plate additive in an agitated bath

As the agitated bath has a high value of heat transfer coefficient, it provides a large amount of convective heat, $h(T_b-T_{mf})$ owing to which a small amount of latent heat of fusion to balance the difference of the conducted heat to the plate and the convective heat available from the bath is required. It is affected by the development of the frozen layer of quite a small thickness [1, 8]. This condition makes the thermal resistance of the thin frozen layer insignificant with respect to the convective thermal resistance of the melt bath [9]. At any instant of time, such a feature sets
in a uniform temperature in the entire frozen layer thickness, $d_m$ which is the freezing temperature, $T_{mf}$ of the bath because the freezing front in contact with the bath always remains at $T_{mf}$. Moreover, the contact interface temperature, $T_e$ between the plate and the frozen layer due to above fact is also at $T_{mf}$. These permit the frozen layer to act as a lump system [9-11] which due to remaining at $T_{mf}$ does not absorb or release sensible heat. The conservation of energy to this lump leads to a balance between the sum of the latent heat of fusion evolved by freezing and convective heat supplied by the bath and the heat conducted to the additive. It takes the form

$$\left(\frac{d\xi}{d\tau}\right)/S_i + B_{im}(\theta_p - 1) = -Q_{mm}, \quad \xi = \xi, \tau > 0 \quad \text{with} \quad BB_{im} = B_i$$  \hspace{1cm} (5.)

It is subjected to an initial condition

$$\xi = 0, \tau = 0$$  \hspace{1cm} (6.)

The conjugated conditions at the interface between the additive plate and the frozen layer can be written as

$$\left(\frac{\partial\theta_i}{\partial\xi}\right)/B = -Q_{mm}, \quad \xi = \xi_m = 0, \tau > 0 \quad \text{with} \quad BB_{im} = B_i$$  \hspace{1cm} (7.)

$$\theta_m = \theta = 1, \quad \xi = \xi_m = 0, \tau > 0$$  \hspace{1cm} (8.)

Note that equations (1) to (8) form the mathematical model of the present problem. This type of model is identified as lump-integral since the frozen layer is assumed to be a lump and the additive is considered an integral system in the direction of heating. Governing equations (1) and (5) assume the uniform but different thermo-physical properties for the frozen layer and the plate additive whereas equations (7) and (8) are written when the surface of the plate is in perfect contact with the surface of the frozen layer with no interface resistance between them. These assumptions in the recent past investigation associated with freezing of the bath material onto the spherical [5, 6] and plate shaped additive [1, 12] having comparative thermal resistances of the bath with respect to the frozen layer provided acceptable solutions. These equations indicate the model dependence upon the independent parameters - initial temperature, $\theta_{ai}$ of the additive, the bulk temperature, $\theta_b$ of the bath, the modified Biot number, $B_{im}$ and the Stefan number, $S_t$. It is noted that the $B_i$ represents the condition of the bath whereas $S_i$ is the indicative of the phase-change parameter of the bath material.

3 Solution

The present model exhibits that it is a moving phase-change problem owing to the presence of equation (5) and is coupled as a result of conjugating conditions, equations (7) and (8). These do not permit the model to give an analytical solution using exact analyses reported in the literature. In such a situation approximate analytical methods become of great significance and practical. One of these methods refer to integral method which yielded simple solutions in closed-form for melting or freezing and cooling or heating problems in previous investigations [13, 14] is applied. Also, in recently investigated other heating and phase-change problems [15-17], this method reduces them to initial value problems, the numerical solutions of which can be readily obtained using standard Runge-Kutta method. In view of this, the governing equation (1) for heating of plate additive has been already expressed in the integral format. For solution, it requires a prior knowledge of temperature field within the heated region of the additive which is assumed to be parabolic.
\[ \theta_a = \theta_{ai} + (1 - \theta_{ai})(1 + \xi_a / \eta)^2 \] (9.)

Note that such a profile is realistic since in the melting [15] and freezing [16] problems it yielded acceptable and accurate results. It satisfies the boundary conditions, equations (3) and (4). Employing equation (9), the integral equation (1) takes the form

\[ \frac{d}{d\tau} \left[ \theta_{ai} \eta + \frac{1 - \theta_{ai}}{3} \right] - \theta_{ai} \frac{d\eta}{d\tau} = \frac{1}{B} \left[ 2(1 - \theta_{ai}) - \left( \frac{\partial \theta_{ai}}{\partial \xi_a} \right)_{\xi_a = -\eta} \right] \]

It reduces to

\[ d \left[ (1 - \theta_{ai}) \eta / 3 \right] / d\tau = 2(1 - \theta_{ai})/(B\eta) \] (10.)

once equation (4) is applied. Similarly, use of equation (9) in the conjugating condition, equation (7) gives.

\[ 2(1 - \theta_{ai})/(B\eta) = -Q_m \] (11.)

Substituting this equation in the energy balance equation (5) leads to

\[ (d\xi / d\tau) / S_i + B_{im} (\theta_b - 1) = 2(1 - \theta_{ai})/(B\eta) \] (12.)

It is noted that equation (12) for the frozen layer thickness, \( \xi \) gets coupled with equation (10) due to presence of \( \eta \) in it. These two equations (10) and (12) form simultaneous differential equation of first order in time, \( \tau \) due to presence of a constant temperature difference \( 1 - \theta_a \) giving a closed-from solution. Their closed-form solutions are often difficult to obtain. However, their examination indicates that equation (10) can be reduced to

\[ d\eta/d\tau = 6/(B\eta) \] (13.)

Satisfying the initial condition, equation (2), it gives

\[ \eta = \sqrt{12\tau/B} \] (14.)

To solve equation (12), it is combined with equation (10) leading to

\[ d \left[ (1 - \theta_{ai}) \eta / 3 - \xi / S_i \right] / d\tau = B_{im} (\theta_b - 1) \] (15.)

It readily gives a closed-form solution

\[ (1 - \theta_{ai}) \eta / 3 - \xi / S_i = B_{im} (\theta_b - 1) \tau \] (16.)

It fulfills the initial conditions, equations (2) and (6). This solution is not an explicit function of \( \tau \), rather functions of \( \tau \) and \( \eta \). To overcome this equation (14) is applied to make equation (16) a function of time.

\[ \xi^* = \xi / S_i = -B_{im} (\theta_b - 1) \tau + (1 - \theta_{ai}) \sqrt{12\tau/B}/3 \] (17.)
Equations (17) and (14) give, respectively, closed-form relations for the growth of the frozen layer, \( \xi \) and the heat penetration depth \( \eta \) in the additive in terms of \( \tau \) measured just after the immersion of the additive in the bath.

### 3.1 Maximum frozen layer thickness and its growth time

To find the development of the maximum frozen layer thickness, \( \frac{d\xi^*}{d\tau} \) found from the equation (17) is made zero.

\[
d\xi^*/d\tau = -B_{im}(\theta_b - 1) + (1 - \theta_{ai})\sqrt{12/B} \tau^{-1/2}/6 = 0
\]

It gives

\[
\tau/B = \left[ (1 - \theta_{ai})/\left( B_{i}(\theta_b - 1) \right) \right]^2 /3 = C_{of}^2 /3
\]

where,

\[
B_{i} = B_{im} B = h b/K_a \quad \text{and} \quad C_{of} = (1 - \theta_{ai})/\left( B_{i}(\theta_b - 1) \right) = \left\{ K_a (T_{mf} - T_{ai})/b \right\}/\left\{ h (T_b - T_{mf}) \right\}
\]

Here, the Biot number, \( B_i \) represents the ratio of the conductive resistance of the additive and the convective resistance of the bath whereas \( C_{of} \), designated as conduction factor, is the ratio of the heat conducted to the additive, \( K_a (T_{mf} - T_{ai})/b \) owing to the difference of the freezing temperature of the bath material and the initial temperature of the additive and the convective heat available from the bath, \( h (T_b - T_{mf}) \). Its values range from 0 to \( \infty \) (0 \( \leq \) \( C_{of} \) \( \leq \) \( \infty \)). Zero indicates preheated additive at the freezing temperature of the bath material permitting no conductive heat transfer to the additive whereas \( \infty \) signifies the bath at the freezing temperature of the bath material resulting non-availability of the convective heat from the bath. Using equation (19), equation (17) provides maximum frozen layer thickness

\[
\xi_{max} = (1 - \theta_{ai})^3 / \left( 3B_{i}(\theta_b - 1) \right) = \left[ (1 - \theta_{ai}) C_{of} \right] /3
\]

since \( d^2\xi^*/d\tau^2 < 0 \), the condition for the maximum value of \( \xi^* \), is satisfied at such a time, which is hereafter called, \( \tau_{max} \).

### 3.2 Total time of freezing and subsequent melting of the bath material, \( \tau_t \)

As in this time, \( \tau_t \), the frozen layer, \( \xi^* \) developed melts completely giving \( \xi^* = 0 \). Its substitution in equation (17) provides an expression for \( \tau_t \)

\[
\sqrt{\tau_t} = \left[ 12/B (1 - \theta_{ai}) \right] / \left[ 3B_{im}(\theta_b - 1) \right]
\]

It is rearranged to provide

\[
\tau_t/B = (4/3) \left[ (1 - \theta_{ai}) / \left( B B_{im}(\theta_b - 1) \right) \right]^2 = (4/3) \left[ (1 - \theta_{ai}) / \left( B_{i}(\theta_b - 1) \right) \right]^2 = (4/3) C_{of}^2
\]

Using equations (22) and (19), the time of melting, \( \tau_m \) of the frozen layer can written as

\[
\tau_m = \tau_t - \tau_{max} = C_{of}^2
\]
Note that equation (19) indicates that the time for the growth of the maximum thickness of the frozen layer $\xi^*_\text{max}$ is always 25% of the total time taken in freezing and its subsequent melting whereas equation (23) states that the melting of the frozen layer takes 75% of the total time.

### 3.3 Condition for the additive to act as semi-infinite

In several practical situations during the period, $\tau_i$ of freezing and melting of the bath material, the additive plate does not get completely heated owing to which the heat does not penetrate the central axis of the plate. In this condition the plate acts as semi-infinite and the heat penetration depth, $\eta$ within it always remains $\eta \leq 1$. Applying it to equation (14) leads to

$$\tau_i/B \leq 1/12$$  \hspace{1cm} (24.)

Substitution of the equation (24) in equation (22) provides

$$C_{oi} \leq 1/4$$  \hspace{1cm} (25.)

It is the requisite condition for the additive to remain semi-infinite during freezing and melting of the bath material.

### 3.4 Reduction of solutions only as a function of time

Since equations (19) to (23) are functions of conduction factor, $C_{oi}$, they including equations (14) and (17) are transformed with respect to this factor reducing them to only a function of time. Using this concept, the growth of frozen layer thickness, equation (17) and the heat penetration depth equation (14) become respectively

$$\xi_c = \xi_c^*/(1-\theta_{ai}) = \sqrt{12\tau_c/3-\tau_c}$$  \hspace{1cm} (26.)

$$\eta_c = \sqrt{12\tau_c}$$  \hspace{1cm} (27.)

where, $\xi_c^* = \xi^*/C_{oi}$, $\tau_c = \tau/(BC_{oi}^2)$, $\eta_c = \eta/C_{oi}$ whereas time of maximum frozen layer thickness equation (19) and total time of freezing and melting equation (22) can be expressed respectively as

$$\tau_c = \tau/(BC_{oi}^2) = 1/3$$  \hspace{1cm} (28.)

$$\tau_{ei} = \tau_i/(BC_{oi}^2) = 4/3$$  \hspace{1cm} (29.)

The growth for the maximum frozen layer thickness, equation (21) is changed to

$$\xi_{c,\text{max}} = \xi_{c,\text{max}}^*/(1-\theta_{ai}) = 1/3$$  \hspace{1cm} where, $\xi_{c,\text{max}}^* = \xi_{c,\text{max}}^*/C_{oi}$  \hspace{1cm} (30.)

Although the freezing and melting of the bath material onto the surface of plate additive follows a complicated heat transfer process, this model enables to make the frozen layer growth, equation (26) and the heat penetration depth, $\eta$ equation (27) only a function of time.

DOI 10.12776/ams.v19i1.87

p-ISSN 1335-1532
e-ISSN 1338-1156
4 Model Validation

To validate the present model, this problem is transformed to heating of the additive by the bath when no freezing of the bath material occurs onto the surface of the plate additive. This can be achieved once the bath is assumed to be highly agitated with heat transfer co-efficient $h \to \infty$, Biot number, $B_i \to \infty$ or the latent heat of fusion of the bath material is very high $L_m \to \infty$, $S_t \to 0$ leading to growth of the frozen layer of insignificant thickness, $\xi \to 0$. The contact interface temperature between the insignificant frozen layer and the plate additive remains at the freezing temperature, $T_{mf}$ of the bath material. These reduce the present problem of heating of the plate at a constant temperature, $T_{mf}$. These concepts vanish equation (17) without altering equation (14) providing

$$\eta = D \tau_h^{1/2}, \text{ where, } D = \sqrt{12} = 3.46$$

The exact solution [18] gives $D = 3.66$ whereas the variational method [19] provided $D = 3.36$. A close agreement is observed validating the present model.

5 Results and Discussions

The mathematical model of lump-integral form just developed for freezing and melting of the bath material around the surface of the plate shaped additive immersed in an agitated bath shows that this occurrence is regulated by the initial temperature, $\theta_{ai}$ of the additive, the bath temperature, $\theta_b$, the bath agitation represented by the Biot number, $B_i$, phase-change parameter of the bath material denoted by the Stefan number, $S_t$ and the property-ratio, $B$ of the additive-bath system. However, in the closed-form solutions $\theta_{ai}$, $\theta_b$, $B_i$, $S_t$ and $B$ appear as a conduction factor, $C_{of}$, equation (20). For different plate shaped additive-bath systems employed in steelmaking and cast iron preparation, values of these parameters appear in Table1. The conduction factor, $C_{of}$ ranges from 0 to $\infty$. $C_{of} = 0$ signifying no conductive heat transfer to the additive which, in turn, does not permit freezing of the bath material onto the additive leading to elimination of the first step of the freezing and melting. In practice it can be achieved by preheating the additive to the freezing temperature of the bath material ($\theta_{ai} = 1$) before its immersion in the bath or increasing the bath agitation for a prescribed temperature, $\theta_{ai}$ of the additive. $C_{of} = \infty$ indicates that the convective heat is not available from the bath. It happens when the bath temperature is maintained at the freezing temperature of the bath material ($\theta_b = 1$). Under this situation, the conductive heat required by the additive after its immersion in the bath is met by only evolution of latent heat of fusion due to freezing of the bath material. Consequently, the frozen layer continues to grow and melting of this layer never takes place. Due to these facts a smaller value of $C_{of}$ close to zero is preferred for growth of a small frozen layer thickness so that much less time is required for freezing with its melting. The Stefan number, $S_t$ is the ratio of sensible heat and latent heat of fusion of the bath material. Its high value represents bath material of small latent heat of fusion leading to growth of large thickness of the frozen layer. In the closed-form solution for the frozen layer thickness it is taken per unit $S_t$, equation (17) termed as $\xi^*$ for making it applicable to any bath material. Further, $\xi^*$ with respect to the conduction factor $C_{of}$ called $\xi_{cm}$ becomes only a function of the initial temperature, $\theta_{ai}$ of the additive, equation (26). The heat penetration depth, $\eta_{cm}$ equation (27) is implicitly dependent on these factors. In these formats, the growth of $\xi_{cm}$ to its maximum thickness takes time $\tau_{cm} = 1/3$ whereas total time $\tau_f$ for freezing to this maximum thickness and its melting is $4/3$. The maximum thickness of the frozen layer assumes $\xi_{cm}^{max} = (1 - \theta_{ai})/3$. These
forms have advantage of applying to any plate shaped additive and the bath materials. For the additive-bath systems of Table 1 with the plate additive of given semi-thickness and initial temperature, total time of freezing and melting along with the maximum thickness of the frozen layer developed can be readily obtained employing equations (22) and (21), respectively.

Table 1 Thermo-physical properties of the Bath materials [1,5,20].

<table>
<thead>
<tr>
<th>Bath material</th>
<th>( K_m )</th>
<th>( \rho_m )</th>
<th>( C_{pm} )</th>
<th>( L_m \times 10^3 )</th>
<th>( T_{mf} )</th>
<th>( T_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast-Iron ~4% C</td>
<td>51.9</td>
<td>7304</td>
<td>417</td>
<td>275.7</td>
<td>1160</td>
<td>1550</td>
</tr>
<tr>
<td>Hot-Metal</td>
<td>35</td>
<td>6850</td>
<td>670</td>
<td>275.7</td>
<td>1150</td>
<td>1500</td>
</tr>
<tr>
<td>Slag</td>
<td>1.06</td>
<td>2890</td>
<td>920</td>
<td>544</td>
<td>1500</td>
<td>1600</td>
</tr>
</tbody>
</table>

Table 2 Thermo-physical properties of the Solid additive [1,5,20].

<table>
<thead>
<tr>
<th>Solid additive</th>
<th>( K_a )</th>
<th>( \rho_a )</th>
<th>( C_{pa} )</th>
<th>( T_{ai} )</th>
<th>( b \times 10^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni</td>
<td>90</td>
<td>8906</td>
<td>449.5</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Ferro-Magnese</td>
<td>7.5</td>
<td>7200</td>
<td>700</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>DRI</td>
<td>2.13</td>
<td>2600</td>
<td>820</td>
<td>20</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3 Non-dimensional parameters of Bath-solid additive system [1,5,20].

<table>
<thead>
<tr>
<th>Bath-solid additive system</th>
<th>( \theta_a \times 10^3 )</th>
<th>( \theta_b )</th>
<th>B</th>
<th>( \theta_{ai} )</th>
<th>( C_{of} )</th>
<th>( S_{i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast iron ~4% C and Ni</td>
<td>17.2</td>
<td>34.5</td>
<td>1.33</td>
<td>0.44</td>
<td>0.33</td>
<td>8.77</td>
</tr>
<tr>
<td>Hot metal and Ferro-Magnese</td>
<td>17.4</td>
<td>34.8</td>
<td>1.3</td>
<td>4.25</td>
<td>4</td>
<td>0.81</td>
</tr>
<tr>
<td>Slag and DRI</td>
<td>13.3</td>
<td>26.6</td>
<td>1.06</td>
<td>0.62</td>
<td>14.08</td>
<td>352.1</td>
</tr>
</tbody>
</table>

*Based on heat transfer co-efficient, \( h = 6000 \text{ Wm}^{-2}\text{k}^{-1} \)

5.1 Effect of conduction factor, \( C_{of} \)

Displayed in Fig. 2 are time variant growth of frozen layer thickness per unit \( S_i \), \( \xi^i \) and the heat penetration depth, \( \eta \) in the additive for a prescribed initial temperature, \( \theta_{ai} \) of the additive. The conduction factor, \( C_{of} \) is taken as a parameter. It indicates that for each \( C_{of} \) the freezing and melting assumes a parabolic behavior with the height of the apex of the parabola signifying the maximum thickness of the frozen layer. Decreasing \( C_{of} \) decreases the height of the apex of the parabola and in turn, the maximum thickness of the frozen layer. Moreover, the apex shifts towards zero time and both the total time of freezing and melting, equation (22) and the time taken to attain the maximum thickness, equation (19) decrease with square of \( C_{of} \). The heat penetration depth at the time of completion of the freezing and melting also diminishes. These findings appear to be realistic since for a prescribed initial temperature, \( \theta_{ai} \) of the additive allowing a certain amount of heat to be conducted to the additive, the reduction in \( C_{of} \) increases...
the availability of convective heat from the bath for a given bath temperature. This increases the Biot number and makes the bath more agitated. Also, as the heat conducted to the additive is sum of the convective heat available from the bath and the latent heat of fusion liberated owing to the freezing of the bath material onto the additive during the development of the frozen layer, increased convective heat from the bath by the reduced $C_{of}$ requires less amount of heat liberated as the latent heat of fusion to meet the above desired conductive heat. It leads to a growth of smaller thickness of the frozen layer, Fig.2.

![Figure 2](image.png)

**Fig. 2** Effect of $C_{of}$, on time dependent frozen layer thickness, heat penetration depth and total time of freezing and melting for additive at $\Theta_{ai}$

5.2 Influence of initial temperature of the additive, $\theta_{ai}$

Illustrated in Fig.3 are the frozen layer thickness, $\xi^*$ and the heat penetration depth, $\eta$ as functions of time for different values of initial temperature, $\theta_{ai}$ of the additive. They are for a particular conduction factor, $C_{of}$. Although the freezing and melting follows the parabolic behavior for each $\theta_{ai}$, with increasing initial temperature, $\theta_{ai}$, the maximum frozen layer thickness, equation (21) decreases, but the time taken to attain this thickness, equation (19), total time of freezing and melting, equation (22) and the heat penetration depth remain unaltered. When the initial temperature, $\theta_{ai}$ of the additive is at the freezing temperature of the bath material, no freezing occurs. The following facts support these predictions. For a specified value of conduction factor, $C_{of}$, decreasing $\theta_{ai}$ increases the heat conduction to the additive and in order to maintain this value of $C_{of}$, the convective heat from the bath is to be increased to the amount by which the heat conduction is increased. As a result, the requirement of latent heat of fusion is increased leading to formation of larger thickness of the frozen layer. Moreover, as the value of $C_{of}$, a ratio of conductive heat transfer to the additive and the convective heat supplied by the bath does not change despite the change in the values of these two heat transfer due to change in $\theta_{ai}$, the requirement of time for development of the frozen layer with its subsequent melting
remains the same. The growth of the frozen layer and its melting, however, gets speed up owing to an increased temperature difference between the bath and the additive once the initial temperature, \( \theta_{ai} \) of the additive is decreased.

![Fig. 3](image)

**Fig. 3** Influence of initial temperature, \( \Theta_{ai} \) of additive on time of growth of frozen layer, total time of freezing and melting and heat penetration depth for a given conduction factor, \( C_{of} \) of bath-additive system

![Fig. 4](image)

**Fig. 4** Time for development of frozen layer, heat penetration depth including total time of freezing and melting. They are for any plate additive having any initial temperature

### 5.3 Growth of frozen layer applicable to any plate additive and bath materials

With respect to the conduction factor, \( C_{of} \), \( \xi^* \) per unit difference of temperature between the freezing temperature of the bath material and the initial temperature, \( \theta_{ai} \) of the additive represented by \( \xi_{cm}^* \), equation (26) and the heat penetration depth denoted by \( \eta_{cm} \) equation (27) become only the function of time. Their plots with time, **Fig.4** assume the behavior similar to those appear in Figs.2 and 3. They act as universal graphs and applied to any plate shaped additive-bath materials provided the freezing and subsequent melting of the bath material onto the additive takes place. In this situation, the maximum frozen layer thickness, \( \xi_{c_{\text{max}}}^* = 1/3 \), equation (30) with its time of formation, \( \tau_{cm} = 1/3 \), equation (28) whereas the total time of freezing of this thickness with its subsequent melting is \( \tau_{ct} = 4/3 \). For any of the additive-bath system contained in Table1, these graphs or equations (30), (28), (29) readily give the
dimensional values of frozen layer thickness, time for its maximum growth and total time of freezing and melting once the data of the Table1 is employed. They are beneficial for industrial applications.

6 Conclusions
A non-dimensional lump-integral model evolved for the first step of the freezing and melting of the bath material onto the surface of the plate shaped additive in an agitated bath gives closed-form solutions for the growth of the frozen layer with its subsequent melting and the heat penetration depth in the plate additive with the time measured immediately after their occurrence. They are regulated by conduction factor, $C_{of}$ and $\theta_{ai}$. Decreasing $C_{of}$ reduces the growth of maximum thickness of the frozen layer, its time of growth and the total time of freezing with its melting. Increasing the initial temperature of the additive, $\theta_{ai}$ the growth of the maximum frozen layer thickness reduces. However, its time of growth and total time of freezing with its melting almost remains the same. This step almost disappears and the frozen layer thickness becomes almost zero once the bath is assumed to be highly agitated $C_{of} \to 0$. With respect to $C_{of}$, these solutions lead to frozen layer and the heat penetration depth only a function of time which are applied to any additive-bath system provided the additive is plate shaped. In this situation, the time taken for the maximum thickness of the frozen layer, $\xi_{c_{max}}$ is $\tau_{c_{max}}=1/3$ whereas the total time taken for the first step is $\tau_{c}=4/3$.

References
Appendix:

b semithickness of the plate, m
B property ratio, \((K_mC_m/K_aC_a)\)
B_i Biot number, \((hb/K_a)\)
B_{im} Modified Biot no., \((hR_0/K_a)\)^\#\((K_aC_a/K_mC_m)\)
C heat capacity \((\rho C_p)\), Jm\(^3\)K\(^{-1}\)
C_p Specific heat, JKg\(^{-1}\)K\(^{-1}\)
C_{of} conduction factor, \(1 - \theta_{ai}/B_i (\theta_b -1)\)
d heat penetration depth or frozen layer thickness at any time, m
h heat transfer coefficient, Wm\(^{-2}\)K\(^{-1}\)
K thermal conductivity, Wm\(^{-1}\)K\(^{-1}\)
L latent heat of fusion, JKg\(^{-1}\)
Q_m heat transfer from the frozen layer to the additive, Wm\(^2\)
Q_{mn} non-dimensional heat transfer from the frozen layer to the additive, \((Q_m/K_a T_{mf})/B\)
S_t Stefan number, \((C_m T_{mf} / L_m \rho_m)\)
t time, s
T temperature, K
x distance along the heat penetration depth or frozen layer, m

Greek letters

\(\alpha\) thermal diffusivity, m\(^2\)s\(^{-1}\)
\(\xi\) non-dimensional thickness of the frozen layer, \((C_md_m/C_a b)\) or distance along frozen layer, \((C_mx_m/C_a b)\) or the additive \((x_a/b)\)
\(\eta\) non-dimensional heat penetration depth in the additive at any time, \((d_a/b)\)
\(\rho\) density, Kg\(^{-1}\)m\(^3\)
\(\theta\) non-dimensional temperature, \((T/T_{mf})\)
\(\tau\) non-dimensional time, \((K_mC_m/C_a^2 b^2)t\)

Subscript

a plate additive
ai initial condition of additive
b bulk condition
e at the interface between the additive and the frozen layer at time.
m frozen bath material