How to Use Confidence Intervals in Selecting a Suitable Time-Dependent Distribution Model for the Process

DOI: 10.12776/qip.v29i1.2095

Milan Terek

Received: 2024-11-08 Accepted: 2024-11-24 Published: 2025-03-31

ABSTRACT

Purpose: This paper proposes the possibility of using confidence intervals for the mean and variance in selecting a suitable time-dependent distribution model for the process.

Methodology/Approach: The approach describes a characteristic under consideration by the distribution, the location, the dispersion, and the shape, all of which are functions of time. The values of the characteristics under consideration are determined by taking samples from the process flow. Time-dependent distribution models are classified into four groups based on whether the location and dispersion moments remain constant or vary over time.

Findings: The paper explains a method for creating random samples, along with the calculation, presentation, and interpretation of the confidence intervals for the mean and variance in choosing the appropriate time-dependent distribution model of the process.

Research Limitation/Implication: The methods described in this document pertain exclusively to continuous quality characteristics. They can be applied to analyse processes across various industrial and economic sectors. The presented procedures assume that all instantaneous distributions are normal.

Originality/Value of paper: Confidence intervals can improve decision-making when selecting a suitable time-dependent distribution model.

Category: Research paper

Keywords: confidence interval for the mean; confidence interval for the variance; time-dependent distribution model; random sample; normal distribution

Research Areas: Quality Engineering

1 INTRODUCTION

Process analysis aims to gain knowledge of a process necessary for effectively controlling and improving the process. This helps ensure that the resulting products meet the required quality standards. A key component of this knowledge is identifying the process distribution and estimating its capability or performance.

Most of the literature concerning the capability or performance of the processes assumes that the process is in a state of statistical control, with stationary, normally distributed processes. Reliable predictions of process performance can only be realised when the process is stable. When the important variables have been identified and the relationship between the important variables and the process output has been quantified, an online statistical process-control technique for process monitoring and surveillance can be employed with considerable effectiveness. Control charts can monitor the process output and detect when input changes are required to return the process to an in-control state (Montgomery, 2013, p. 15).

The procedure is based on the idea that there are two distinct sources of variability. The common (chance) causes are embedded in the system or process itself, while assignable (special) causes usually arise from an external source. A process that operates with only common causes of variation present is said to be in statistical control (in-control state). A process that operates in the presence of common and assignable causes is an out-of-control process (in an out-of-control state). Chakraborty and Graham (2019, p. 24) state that in a manufacturing process, for example, assignable causes can typically arise from defective raw materials, improperly adjusted or controlled machines, or operator errors. Oakland and Oakland (2019) state that the assignable causes can be people, plants/equipment, processes/procedures, materials, or the environment. Any identified assignable causes are addressed by engineering and operating personnel to eliminate them. Once removed, the process will transition from an out-of-control to an in-control state.

Unfortunately, in many processes, the assumption that the process in a state of statistical control is normally distributed with constant parameters, which can change only by the influence of known assignable causes, may not be met. The first possible problem is recognising assignable causes and their connection with the patterns in control charts. According to Oakland and Oakland (2019, p. 316), numerous potential assignable causes exist for processes being out of control. It is complicated, even dangerous, to try to find an association between types of causes and patterns shown on control charts. Many causes could give rise to different patterns in different industries and conditions. Thus, recognizing the assignable causes that cause this state can be very challenging when a point appears outside control limits in a control chart.

Additionally, it is possible that we do not have a complete list of assignable causes for our process. Oakland and Oakland (2019, p. 317) state that providing a complete list of assignable causes for a specific process is practically impossible.

When used carefully, the control chart informs us when to look for trouble. This typically contributes 10 – 20 percent of the problem. The bulk of the work in making improvements is associated with finding where to look and which causes are operating. It follows from the above that when the process is out of control, it is often difficult to identify the assignable causes responsible for it. Consequently, we cannot take corrective actions to bring the process to an in-control state. Also, the common causes may change over time; thus, the process distribution may change, even without assignable causes.

In ISO 22514-2:2017 (ISO, 2017), it is stated that a comprehensive analysis of production processes shows that over time, it is very rare for processes to remain in a stationary state. Therefore, it is useful to analyse the process behaviour over time. An appropriate method of statistical process control and process performance assessment can only be chosen based on the results of such analysis. Time-dependent distribution models are used to analyse the behaviour of a characteristic over time.

This paper will investigate how confidence intervals for the mean and variance can be used to estimate time-dependent distribution models. We will prove that if at least two confidence intervals for the mean do not overlap or just touch, it indicates a change in the mean over time. Under certain conditions, the same is also true for the confidence intervals for the variance. We will use simulations to illustrate this concept. We aim to demonstrate that confidence intervals are useful for studying changes in a characteristic's location and variability over time. Applying the proposed approach for estimating a time-dependent distribution model can lead to a more accurate and reliable characterisation of the process, resulting in better decision-making on the process.

We begin by categorising time-dependent distribution models, then we address the proofs related to the overlapping of confidence intervals, and finally, we demonstrate the obtained results through simulations.

2 LITERATURE REVIEW

The instantaneous (short time) distribution describes the behaviour of the characteristic during a short time interval, typically when a sample is taken from the process. The process distribution when a process is observed continuously over a longer period is called the resulting distribution. This distribution is described by a time-dependent model that reflects the instantaneous distribution of the characteristic being observed and the changes in location, dispersion, and shape parameters during the time interval of process observation. In practice, the resulting distribution can be represented by the entire dataset or all subgroups obtained during the process observation period (ISO, 2017, p. 3).

Time-dependent distribution models can be classified into four groups according to whether the location and dispersion moments are constant or changing. A process with constant location and dispersion over time is categorised as a time-

dependent distribution model A. In this case, all the means and variances of the instantaneous distributions are equal to each other, and they are equal to the resulting distribution. If the dispersion of a process changes over time while the location remains constant, it falls under time-dependent distribution model B. When the dispersion is constant but the location changes, it is classified as a time-dependent distribution model C. Otherwise, it is categorised as a time-dependent distribution model D. The distribution model A has two alternatives − A1 with normal and A2 with not normal but unimodal instantaneous distributions. The distribution model B is characterised by systematic and random (e.g., lot-to-lot) change of dispersion, normal instantaneous and unimodal resulting distribution. The distribution model C has four variants − C1 with a random change of location, a normal instantaneous distribution, and a normal resulting distribution; C2 differs from C1 by having a resulting distribution that is not normal but unimodal; C3 is characterised by a systematic (trend) change of location, unimodal instantaneous distributions and, the resulting distribution of any shape; C4 indicates the process model with systematic and random (e. g. lot to lot) change of location, an instantaneous distribution of any shape, and the resulting distribution of any shape. The process model D models the process with a systematic and random change of location and dispersion and instantaneous and resulting distribution of any shape (ISO, 2017; Jarošová and Noskievičová, 2015; Zgodavová, et al., 2020). The method for calculating capability and performance indices varies based on the time-dependent distribution model used for the process.

The process analysis needs to identify the appropriate time-dependent distribution model. Certain models suppose the normal distribution for the characteristic being studied. The graphical method enables the visual assessment of normality (see Normality test). Normality tests are additional tools used in conjunction with other statistical methods. They are used to determine whether a normal distribution can accurately represent a population. Several tests are available for this purpose (for a list, see Ghasemi and Zahediasl, 2012; Thode, 2002).

When the normality of instantaneous distributions is not rejected, we can test the equality of means of the instantaneous distributions using the analysis of variance (ANOVA). Three assumptions are necessary to use ANOVA: For each population, the dependent variable is normally distributed, the variance of the dependent variable is the same for all populations, and the observations must be independent (Anderson et al., 2020; Terek, 2017). When the null hypothesis is rejected in the ANOVA, it only leads to the conclusion that the means are not all equal. To identify where differences between the means occurred, Fisher's LSD (Least Significant Difference) procedure can be utilised. This involves performing a *t*-test for all pairs of mean values at the same level of significance.

The assumption that variances are equal across all populations can be tested using Bartlett's Test (Statistics How To, 2024). Bartlett's test is sensitive to departures from normality, meaning that if the samples come from non-normal distributions, then Levene's and the Brown-Forsythe test are alternative tests that are less sensitive to departures from normality. If the null hypothesis of equal variances is

rejected in these tests, it is concluded that there are differences among the variances.

3 METHODOLOGY

The values of the considered characteristics are typically determined based on samples taken from the process flow. The sample size and frequency should be chosen depending on the type of process and the type of product to ensure that all important changes are detected in time (ISO, 2017, p. 3). Therefore, determining the type of time-dependent distribution model will be based on multiple samples from the process flow over time, covering all possible expected changes in the process.

Let us understand how to take the samples. A population in which it is impossible or unrealistic to record every unit in real-time is considered infinite even when it is finite. A random sample from an infinite population is obtained by selecting n units to satisfy two conditions: each selected unit is from the same population, and each unit is selected independently. Then, the observations are statistically independent and identically distributed random variables, and the usual statistical inference methods can be used. The sample should consist of units produced simultaneously or as closely together as possible to ensure that all observations are from the same population (probability distribution). Ideally, consecutive units of production should be taken. The independence condition should be fulfilled so that the units are produced independently. Thus, the production of each unit can be considered the implementation of an independent random experiment (Terek, 2023). Suppose we take more random samples from the process flow over a time that covers all potential expected changes. The behaviour of the parameters in instantaneous distributions will be studied based on these samples.

In the first step of the analysis, the normality of the distributions must be checked. Usually, about 20 points are required to produce normal probability plots that are stable enough to be easily interpreted (Montgomery et al., 2021, p. 144). We recommend a sample size of n = 30 for all samples to ensure that the confidence interval for the mean is applicable even if the population distribution is not normal.

In the upcoming analysis, we will calculate confidence intervals. Assuming that the hypothesis regarding normal distribution is not rejected based on either sample, we can consider all instantaneous distributions to be normal. After this, we can calculate the $(1 - \alpha)100\%$ confidence interval for the mean based on each sample. If $\bar{x}$ is the value of the sample mean of a random sample of size n from a normal population with the known variance σ^2, then

$$\bar{x} - z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \qquad (1)$$

where $z_{1-\frac{\alpha}{2}}$ is $(1-\frac{\alpha}{2})100\%$ quantile of standard normal distribution and $\sigma = \sqrt{\sigma^2}$ is standard deviation, is a $(1 - \alpha)100\%$ confidence interval for the mean of the

population. Since all taken samples are of the size n = 30, we can estimate σ by s, where $s = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^2}$ and replace σ in relation (1) by s.

If the normality of all instantaneous distributions was confirmed, the confidence intervals for variance can be used to check the variability of the process. If s^2 is the value of the sample variance of a random sample of size n from a normal population, then

$$\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}(n-1)} \tag{2}$$

where $\chi^2_{\frac{\alpha}{2}}(n-1)$ and $\chi^2_{1-\frac{\alpha}{2}}(n-1)$ are the $\frac{\alpha}{2}100\%$, and $\left(1-\frac{\alpha}{2}\right)100\%$ quantile of a chi-square distribution with $(n-1)$ degrees of freedom is a (1 – α)100% confidence interval for σ^2. Finally, we obtain the time series of confidence intervals for the mean and for the variance in the whole period that covers all possible expected changes in the process.

4 CONFIDENCE INTERVALS AND CHANGING THE PARAMETERS OVER TIME

Suppose we have taken more independent random samples of size $n = 30$ from the process flow over a time that covers all potential expected changes, and all instantaneous distributions are normal. Confidence intervals for the mean and the variance were computed based on these samples. Now, let us consider a scenario where there are two non-overlapping or just touching confidence intervals for the mean within the sequence of these intervals over time:

$$\bar{x}_1 - z_{1-\frac{\alpha}{2}}\frac{s_1}{\sqrt{n}} \leq \mu_1 \leq \bar{x}_1 + z_{1-\frac{\alpha}{2}}\frac{s_1}{\sqrt{n}}$$

$$\bar{x}_2 - z_{1-\frac{\alpha}{2}}\frac{s_2}{\sqrt{n}} \leq \mu_2 \leq \bar{x}_2 + z_{1-\frac{\alpha}{2}}\frac{s_2}{\sqrt{n}}$$

and

$$\bar{x}_1 - z_{1-\frac{\alpha}{2}}\frac{s_1}{\sqrt{n}} \geq \bar{x}_2 + z_{1-\frac{\alpha}{2}}\frac{s_2}{\sqrt{n}} \tag{3}$$

After the simple adjustments of (3), we get

$$\bar{x}_1 - \bar{x}_2 \geq z_{1-\frac{\alpha}{2}}\frac{s_1+s_2}{\sqrt{n}} \tag{4}$$

Now, based on the same two samples, we want to conduct a test:

$$H_0: \mu_1 = \mu_2 \tag{5}$$

$H_1: \mu_1 \neq \mu_2$

α – significance level of the test.

Since both sample sizes are large ($n = 30$) and we do not know the variances of the populations, we can replace the population variances σ_1^2 and σ_2^2 with the values of the sample variances $s_1^2$and s_2^2 in the definition of the critical region of the test, which is then

$$|z| > z_{1-\frac{\alpha}{2}}, \text{ where } z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2+s_2^2}{n}}}$$

If $\bar{x}_1 > \bar{x}_2$, then

$$\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2 + s_2^2}{n}}} > z_{1-\frac{\alpha}{2}}$$

After a simple adjustment, we get

$$\bar{x}_1 - \bar{x}_2 > z_{1-\frac{\alpha}{2}}\sqrt{\frac{s_1^2+s_2^2}{n}} = z_{1-\frac{\alpha}{2}}\frac{\sqrt{s_1^2+s_2^2}}{\sqrt{n}} \quad (6)$$

We can prove that $s_1 + s_2 > \sqrt{s_1^2 + s_2^2}$:

$$\sqrt{s_1^2 + s_2^2} = \sqrt{(s_1 + s_2)^2 - 2s_1 s_2}$$

and

$$s_1 + s_2 = \sqrt{(s_1 + s_2)^2}$$

From the comparison of the last two relations, it follows that:

$$s_1 + s_2 > \sqrt{s_1^2 + s_2^2} \quad (7)$$

It follows from the validity of (7) that if relation (4) holds, then relation (6) must also apply. This indicates that if the relation (3) is true, the value of the test statistic falls within the critical region of the test. Therefore, if the two intervals do not overlap or just touch each other, in test (5) at the significance level α, we reject H_0 in favour of H_1. Consequently, if there are at least two confidence intervals in the time series that do not overlap or just touch each other, we can conclude that the mean of the process changes over time.

Now, let's consider there are two non-overlapping or just touching confidence intervals for the variance within the sequence of these intervals over time:

$$\frac{(n-1)s_1^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} \leq \sigma_1^2 \leq \frac{(n-1)s_1^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}; \qquad \frac{(n-1)s_2^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} \leq \sigma_2^2 \leq \frac{(n-1)s_2^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}$$

and

$$\frac{(n-1)s_1^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} \geq \frac{(n-1)s_2^2}{\chi^2_{\frac{\alpha}{2}}(n-1)} \quad (8)$$

After the simple adjustments of (8), we get

$$\frac{s_1^2}{s_2^2} \geq \frac{\chi^2_{1-\frac{\alpha}{2}}(n-1)}{\chi^2_{\frac{\alpha}{2}}(n-1)} \tag{9}$$

Based on the same two samples, we want to conduct a test:

$$H_0: \sigma_1^2 = \sigma_2^2 \tag{10}$$

$$H_1: \sigma_1^2 \neq \sigma_2^2$$

α – significance level of the test.

The critical region of the test is

$$\frac{s_1^2}{s_2^2} > F_{1-\frac{\alpha}{2}}(n-1, n-1) \text{ for } s_1^2 > s_2^2 \tag{11}$$

where $F_{1-\frac{\alpha}{2}}(n-1, n-1)$ is the $\left(1-\frac{\alpha}{2}\right)100\%$ quantile of a Fisher distribution with $(n-1)$ and $(n-1)$ degrees of freedom. We can easily show that for $n = 30$, and usual significance levels α = 0.01; 0.05; 0.1, condition (12) is met:

$$\frac{\chi^2_{1-\frac{\alpha}{2}}(n-1)}{\chi^2_{\frac{\alpha}{2}}(n-1)} > F_{1-\frac{\alpha}{2}}(n-1, n-1) \tag{12}$$

It follows from the validity of condition (12) that if relation (9) holds, then relation (11) must also apply. This indicates that if the relation (8) is true, the value of the test statistic falls within the critical region of the test. That means that if the two intervals do not overlap or just touch each other, in test (10) at the level of significance α we reject H_0 in favour of H_1. Consequently, if there are at least two confidence intervals in the time series that do not overlap or just touch each other, we can conclude that the variance of the process is changing over time. If the values n and α are different of $n = 30$, α = 0.01; 0.05 or 0.1, the fulfilment of the relation (12) has to be verified.

If all confidence intervals for the mean overlap, it does not necessarily indicate that the mean is not changing over time. In such cases, conduct tests (5) for the pairs of means or apply ANOVA. Similarly, if all confidence intervals for the variance overlap, it does not mean the variance is not changing over time. In this situation, perform tests (10) for the pairs of variances or use Bartlett's test.

In general, confidence intervals can be beneficial in cases where the normality of all instantaneous distributions was not rejected, and there are non-overlapping or just touching confidence intervals. Then, they can significantly quicken and make the recognition of the time-dependent distribution models A1, B, C1, and C2 more accurate. The confidence intervals can eventually also help identify assignable causes and indicate trends, if any. However, if all confidence intervals in each diagram overlap, further analysis should be conducted using statistical tests.

4.1 Simulation experiments

In our analysis, we will only consider cases where the acceptance of normal distribution applies to instantaneous distributions. This applies to models A1, B, C1, and C2. We will consider a characteristic: the dimension of a product measured in millimetres (mm).

In situation A1, we assume that all instantaneous distributions have a normal distribution with a mean of $\mu = 100$ mm and a standard deviation of $\sigma = 0{,}01$ mm. Using Excel, we generated ten random samples of size 30 from this distribution. For each sample, we calculated the 95% confidence interval for the mean and the 95% confidence interval for the variance. We have included a diagram showing 300 observations (Figure 1 on the left) and time series diagrams of the confidence intervals for the mean (Figure 1 on the right) and for the variance (Figure 2).

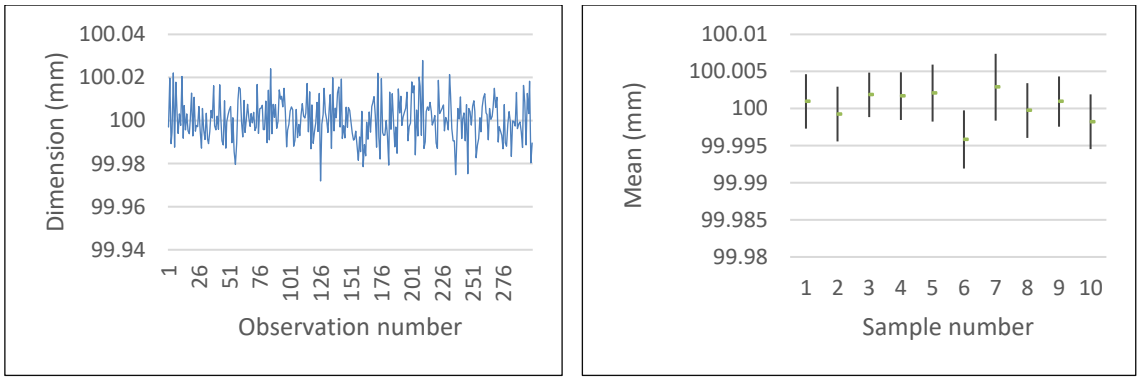

Figure 1 – A1: Observations and confidence intervals for the mean

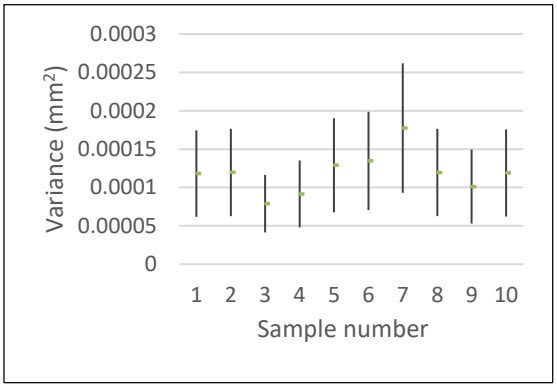

Figure 2 – A1: Confidence intervals for the variance

In situation A1, the confidence interval diagrams (see Figures 1 and 2) show that all intervals overlap. This suggests that we have not proven the differences in means and variances of the instantaneous distributions. However, it does not necessarily mean that they are equal. In these situations, conduct tests (5) for the pairs of means and tests (10) for the pairs of variances. Alternatively, further analysis can be conducted using ANOVA and Bartlett's test.

In model situation B, we assume that all instantaneous distributions have a normal distribution with a mean of $\mu = 100$ mm and that the standard deviation of the instantaneous distributions varies randomly from 0.01 to 0.02 mm. We generated 10 random samples of size $n = 30$ from these distributions. For each sample, we calculated a 95% confidence interval for the mean and a 95% confidence interval for the variance. We have included a diagram showing 300 observations (Figure 3 on the left) and time series diagrams of the calculated confidence intervals (for the mean – Figure 3 on the right and for the variance – Figure 4).

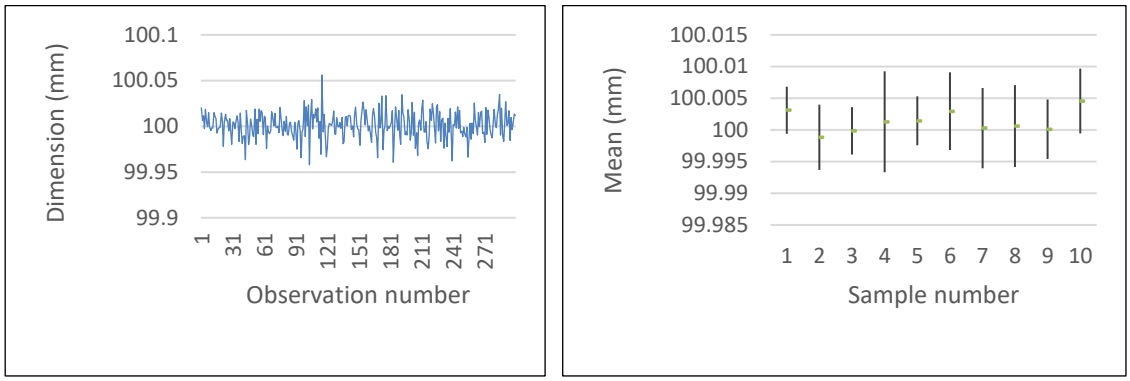

Figure 3 – B: Observations and confidence intervals for the mean

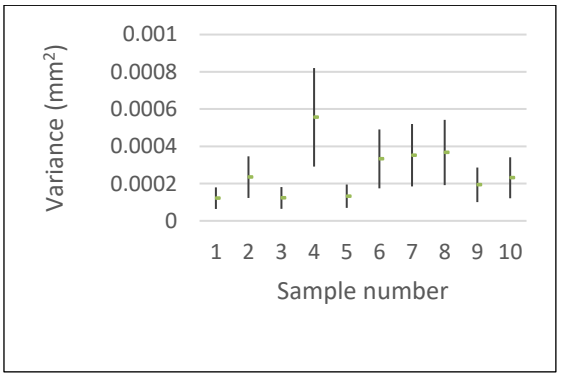

Figure 4 – B: Confidence intervals for the variance

In scenario B, the confidence intervals diagram for the variance shows non-overlapping intervals (see Figure 4), indicating that there is statistical evidence of variance changes over time. In contrast, the confidence intervals diagram for the mean shows that all intervals overlap (refer to Figure 3 on the right). This suggests that we have not proven differences in the means of instantaneous distributions. However, this does not necessarily imply that the expected value is not changing over time. Tests (5) should be conducted for the pairs of means or to utilise the ANOVA test.

In the model situation C1 and C2, we assume that all instantaneous distributions have a normal distribution with a standard deviation of 0.01 mm, and the mean varies randomly from 99.985 mm to 100.015 mm. We generated 10 random samples of size $n = 30$ from these distributions. For each of them, we calculated

a 95% confidence interval for the mean and a 95% confidence interval for the variance. We have included a diagram with 300 observations (Figure 5 on the left and time series diagrams of the calculated confidence intervals (for the mean - Figures 5 on the right and for the variance – Figure 6).

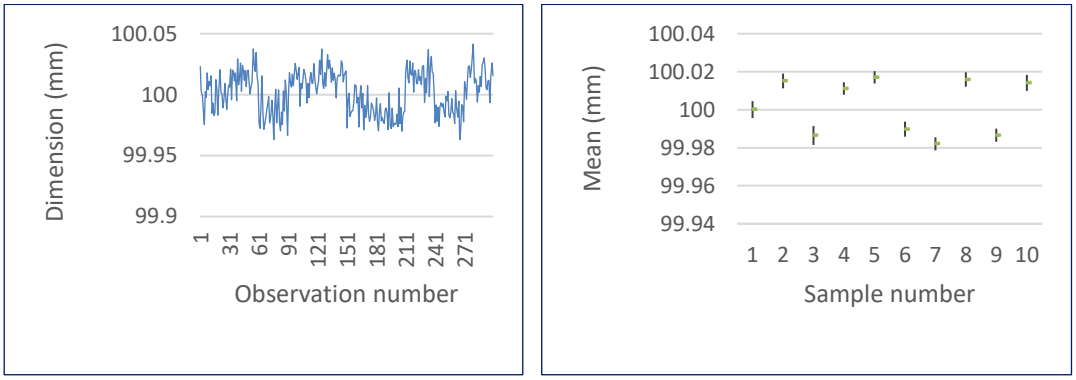

Figure 5 – C1, C2 – Observations and confidence intervals for the mean

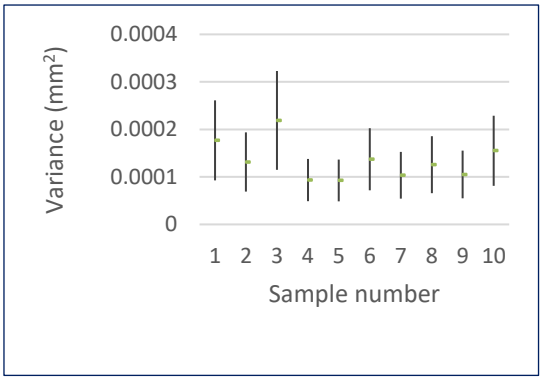

Figure 6 – C1, C2 – Confidence intervals for the variance

In cases C1 and C2, the confidence intervals diagram for the mean shows non-overlapping intervals. This indicates that we have statistical evidence that the mean varies over time (see Figure 5 on the right). In contrast, the confidence intervals diagram for the variance (see Figure 6) shows that all intervals overlap. This suggests that we have not proven differences in the variances of the instantaneous distributions. However, this does not imply that the variance remains constant over time. To investigate this further, we should conduct tests (10) for the pairs of variances, or we could use Bartlett's test for additional analysis.

5 CONCLUSION

The paper proposes possibilities for estimating a time-dependent distribution model for the process using confidence intervals for the mean and variance. It assumes a normal distribution of all instantaneous distributions. The time-dependent distribution model used for the process determines the method for calculating capability and performance indices.

This paper recommends displaying the time series of confidence intervals for the mean and variance of all instantaneous distributions in separate diagrams. This will give an instant idea of the situation (see figures in section 4.1). We have proven that if at least two confidence intervals for the mean do not overlap or just touch each other, it indicates a change in the mean of the process over time. Similarly, for the variance, if there are at least two non-overlapping or just touching confidence intervals, it suggests a change in the variance of the process over time (if condition (12) is met). When all intervals in each diagram overlap, it does not provide evidence of changing parameters over time. However, this does not mean that the parameters are constant over time. In such cases, tests (5) and/or (10) should be conducted. Alternatively, further analysis can be conducted using ANOVA and/or Bartlett's test.

The proposed analysis method can rapidly determine whether at least two confidence intervals do not overlap or just touch. It will provide the basis for the corresponding decision on changing the parameter over time. The diagrams can also indicate any trends in the parameter's development, if any, and eventually help identify assignable causes. All these possibilities are illustrated in section 4.1. In Nussbaum (2024), it is noted that statistical tests should be supplemented with other methods, such as graphical control. Therefore, it is also recommended that diagrams of observations be included in the analysis. In general, confidence intervals can be understood as a supplementary tool to the relevant statistical tests, which can even replace them in the case of non-overlapping intervals. Applying the proposed procedure for estimating a time-dependent distribution model can lead to a quicker, more accurate, and more reliable characterisation of the process, resulting in better decision-making. In future research, the use of confidence intervals for the mean for estimating a time-dependent distribution model could be explored when the instantaneous distributions are not normal.

ACKNOWLEDGEMENTS

The author expresses gratitude to the Vysoká škola manažmentu/City University of Seattle for its comprehensive support of his research activities.

REFERENCES

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., Cochran, J. J., Fry, M. J. and Ohlmann, J. W., 2020. *Statistics for Business and Economics. 14e Edition.* Boston: Cengage Learning, Inc.

Bartlett's Test, 2024. Definition and Examples. [online] Available at https://www.statisticshowto.com/bartletts-test/ [21 May 2024].

Chakraborty, S., Graham, M. A., 2019. *Nonparametric Statistical Process Control*. Hoboken: John Wiley & Sons Ltd.

Ghasemi, A., Zahediasl, S., 2012. Normality Tests for Statistical Analysis: A Guide for Non-Statisticians. *International Journal of Endocrinology and Metabolism*, 10 (2). https://doi.org/10.5812/ijem.3505.

ISO, 2017. ISO 22514-2 Statistical methods in process management – Capability and performance – Part 2: Process capability and performance of time-dependent process models. Electronic documents. Geneva: ISO.

Jarošová, E., Noskievičová, D., 2015. *Pokročilejší metody statistické regulace procesu.* Praha: Grada Publishing.

Montgomery, D. C., 2013. *Introduction to Statistical Quality Control. Seventh edition.* Hoboken: J. Wiley and Sons.

Montgomery, D. C., Peck, E. A., Vining, G. G., 2021. *Introduction to linear regression analysis. Sixth edition.* Hoboken: John Wiley & Sons.

Nussbaum, E. M., 2024. *Categorical and nonparametric data analysis: choosing the best statistical technique. Second edition.* New York: Routledge.

Oakland, J., Oakland, R., 2019. *Statistical Process Control. 7th Edition*. New York: Routledge.

Statistics How To, 2024. Bartlett's Test for Homogeneity of Variances: Definition and Examples. [online] Available at https://www.statisticshowto.com/bartletts-test/ [21 May 2024].

Terek, M., 2017. *Interpretácia štatistiky a dát. 5. doplnené* vydanie [Interpretation of statistics and data. 5th revised ed.]. Košice: Equilibria.

Terek, M., 2023. How to Estimate the Sigma Level of the Process. *Quality Innovation Prosperity*, 27(3).

Thode, H.C. Jr., 2002. *Testing for Normality*. New York: Marcel Dekker.

Zgodavová, K., Bober, P., Majstorovič, V., Monková, K., Santos, G., Juhászová, D., 2020. Innovative Methods for Small Mixed Batches Production System Improvement: The Case of a Bakery Machine Manufacturer. *Sustainability* 2020, 12, 6266. https://doi.org/10.3390/su12156266.

ABOUT THE AUTHOR

Milan Terek ORCID: 0000-0001-5638-9287 (M.T.) – Professor, School of Management, Bratislava, Slovak Republic, e-mail: milan.terek1@gmail.com; mterek@vsm.sk.

CONFLICTS OF INTEREST

The authors declare no conflict of interest. The funders had no role in the design of the study, in the collection, analyses, or interpretation of data, in the writing of the manuscript, or in the decision to publish the results.