Innovative Quick-switching Sampling System for Product Quality Sentencing Integrated with Process Incapability Index C_{pp}

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ABSTRACT

Purpose: This study develops the novel inspection concept of quick switch sampling (QSS), integrating the process incapability index to determine the quality of the product based on the nonconforming fraction.

Methodology/Approach: Product quality acceptance is determined through a nonlinear optimisation model that minimises the average sample number (ASN) for inspection, subject to constraints on predetermined quality levels and risks. The plan's effectiveness is evaluated based on the efficiency of ASN and the discriminatory power of its operating characteristics (OC) curve.

Findings: The findings reveal that the QSS sampling strategy provides requisite quality assurance with reduced sample sizes relative to the conventional quality inspection model while maintaining discriminatory power between business partners. These findings highlight the potential of QSS systems to enhance the effectiveness of quality control while maintaining stringent quality standards.

Research Limitation/Implication: The study was conducted under the assumption that quality characteristics are normally distributed.

Originality/Value of paper: The development of a QSS plan integrated with the process incapability index offers adaptive inspection protocols that dynamically adjust inspection stringency in response to fluctuations in product quality and accommodate product sensitivity in terms of process accuracy and precision.

Category: Research paper

Keywords: quality control and assurance; normal inspection; process incapability index; quick switching rules; tightened inspection; optimisation

Research Areas: Quality Engineering; Quality Management.

1 INTRODUCTION

Acceptance sampling plans are effective and valuable instruments incorporated into statistical quality control (SQC) that assist manufacturers and consumers in deciding if a product batch or lot ought to be accepted or rejected (Schilling, 1982) (Darmawan, Bahri, *et al.*, 2025). The main aim of a sampling plan is to offer decision-makers a systematic approach for deciding the fate of product lots according to established quality and risk standards (Montgomery, 2019). The effectiveness of a sampling plan can be evaluated using the operating characteristic (OC) curve and average sample number (ASN). The OC curve shows the likelihood of acceptance at different quality levels, highlighting a sampling plan's ability to differentiate. A steeper OC curve demonstrates greater discriminatory capability, allowing for more precise distinction between acceptable and unacceptable product lots. Moreover, a sampling system may include various plans with transition rules, enabling the monitoring of inspection records and the enhancement of sampling plan efficiency. These switching rules enable adaptive inspection protocols, using standard inspection plans for products showing high quality, while stricter inspection plans are adopted when there are notable decreases in product quality. Through the combination of various sampling plans alongside switching rules, organisations can improve the accuracy and effectiveness of their quality control measures, thereby guaranteeing the provision of superior products that adhere to strict quality criteria (Darmawan, Wu, *et al.*, 2025). This method allows industries to adapt quickly to variations in product quality, thus reducing the chance of defective products (Wu & Darmawan, 2025).

The normal-tightened-normal sampling method, often known as quick switching systems (QSS), is a sampling approach that flexibly modifies inspection rigour based on variations in product quality. The origin of this rapid sampling method can be linked to MIL-STD-105D, with key input from Dodge (1965). Hald & Thyregod (1965) suggested normal and tightened inspection sampling by attributes. Romboski (1969) performed a thorough examination of the system's characteristics using two different process models (QSS(n, c_N, c_T)) and later suggested essential recommendations for successful execution. Soundararajan & Arumainayagam (1990) advanced this idea by creating improved versions of QSS that included master tables. Later studies by Govindaraju & Ganesalingam (1998) proposed a two-plan sampling method featuring a zero acceptance number for inspection, enabling a reduction in sample size while preserving discrimination capability.

The variable quick switching sampling (VQSS) technique was developed by Soundararajan & Palanivel (2000) for quality characteristics with double specification limits and normal distribution. This QSVSS system employs a fixed sample size but different critical values for normal and tightened inspections. This VQSS scheme is designed for normally distributed data with double specification limits, utilising a fixed sample size (n) for both normal and tightened inspections, but with distinct critical values (k_T and k_N). Building on this, Balamurali & Usha (2012) developed a VQSS model incorporating dual specification limits. Further expanding on this work, Balamurali & Usha (2014) enhanced the VQSS scheme by integrating the process capability index, thereby increasing its effectiveness and applicability across various industrial contexts.

Research on sampling plans that include capability indices has progressed, leading to the enhanced development of the VQSS scheme to incorporate different capability indices. Liu & Wu (2016) notably integrated the S_{pk} index into VQSS, whereas Wu et al. (2017) introduced two variants of (($n;\ k_{\mathrm{N}}$, k_{T}) and (n_{N}, n_{T}; k)) that employed the C_{pk} index. Furthermore, Balamurali & Usha (2017)have played a role in this field by incorporating the C_{pmk} index into VQSS($n;\ k_{\mathrm{N}}$, k_{T}), illustrating the continuous advancement of this approach. These groundbreaking studies have established a foundation for creating advanced QSS systems, allowing industries to enhance their quality control methods and respond flexibly to variations in product quality. By utilising these sophisticated sampling methods, organisations can improve the effectiveness of their quality control procedures.

Extensive research has been carried out in recent years to enhance VQSS from various perspectives, with inputs from Wang et al. (2021), Wang (2022), Wang & Shu (2023), Liu et al. (2023), Wu et al. (2024), Wang et al. (2025), and Wang & Wu (2025). Typically, VQSS tends to depend on process yield and high yield to assess product quality; however, this method overlooks fluctuations that occur within specification limits. The process incapability index (C_{pp}) provides a clearer view of process performance by highlighting the distinct difference between process accuracy and precision. Thus, this research seeks to develop two main types of VQSS for double sample size and single critical value ((n_{N}, n_{T}; k) and (n_{N}, mn_{N}; k)) employing the C_{pp} index, as well as to investigate, examine, and compare their performance and efficiency. Consequently, the creation of a VQSS strategy combined with the process incapability index provides flexible inspection protocols that modify inspection rigour according to variations in product quality and consider product sensitivity regarding process accuracy and precision.

2 PROCESS INCAPABILITY INDEX C_{PP}

Process capability indices (PCIs) serve as valuable instruments for evaluating process capability, as their formulas are simple to grasp and easy to implement. The quadratic loss function index, C_{pm}, was created to evaluate and track the performance of production floors using the quality loss function. Nonetheless, due to the analytical complexity of the C_{pm} index's statistical characteristics, Greenwich and Jahr-Schaffrath (1995) advocated for using the process incapability index, C_{pp}, to assess production process performance, as it distinctly differentiates between process accuracy and precision. The C_{pp} index can be expressed as $C_{pp} = C_{ai} + C_{pi}$, where C_{ai} reflects the process inaccuracy index (departure process mean from target value T) and C_{pi} (imprecision index) measure the extent of accuracy and precision in the process, respectively. As a result, the C_{pp} index can be represented as:

$$C_{pp} = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2 \tag{1}$$

where μ indicates the average of the process, σ indicates the standard deviation of the process, T represents the target value, D=d/3, with d defined as (USL−LSL)/2, the half-width of specification, and USL and LSL denote the upper and lower specification limits, respectively. Furthermore, C_{ai} is computed as $(\mu-T)^2/(D)^2$, while C_{pi} is represented as $\sigma^2/(D)^2$. The index C_{ai} assesses the deviation of process mean m from the target value T. For a centred process, C_{ai}=0 indicates that the process is perfectly on target; C_{ai} = 1 signifies that $\mu = USL$, and C_{ai} = -1 signifies $\mu = LSL$. The index C_{ai} gauges the extent of process departure ratios, whereas the index C_{pi} evaluates the scale of process variation. The C_{pp} index is a simple alteration of the C_{pm} index, providing a clear differentiation between data concerning process accuracy and process precision. Consequently, C_{pp} is a preferred option for engineers evaluating process potentials and performance. Table 1 shows a collection of commonly used C_{pp} values with their corresponding C_{pm} values (Chen & Chen, 2008).

Table 1 – Commonly used C_{pp} and their corresponding C_{pm} values

Condition	C_{pp}	C_{pm}
Incapable	4	0.50
Capable	1	1.00
Satisfactory	0.5653	1.33
Good	0.4444	1.50
Excellent	0.3586	1.67
Super	0.25	2.00

Given that population parameters are often unknown in real-world applications, statistical inference relies on sample data to estimate these parameters for the C_{pp} index. A natural estimator, $\hat{C}_{pp}$, is therefore employed to assess process capability through the C_{pp} incapability index. The maximum likelihood estimate (MLE) of C_{pp}, serves as a basis for this estimation (Chen, 1998):

$$\hat{C}_{PP} = \left(\frac{(\bar{X}-T)^2}{D^2}\right) + \frac{S_n^2}{D^2} = \left(\frac{(\bar{X}-T)^2}{D^2}\right) + \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i-\bar{X})^2}{D^2} = \frac{\sum_{i=1}^{n}(X_i-T)^2}{nD^2}. \tag{2}$$

where $\bar{X} = \sum_{i=1}^{n} X_i / n,\ S_n^2 = \sum_{i=1}^{n}(X_i-\bar{X})^2 / n.$

Building on Equation 2 and the premise that the data follows a normal distribution, it is possible to establish a mathematical correlation between the C_{pp} index and its estimator, which can be formulated as follows:

$$\frac{\hat{C}_{PP}}{C_{PP}} = \frac{\sum_{i=1}^{n}(X_i-T)^2}{nD^2} \times \frac{D^2}{\left[\sigma + (\mu-T)^2\right]} = \frac{\dfrac{\sum_{i=1}^{n}(X_i-T)^2}{\sigma^2}}{n\left[1+\left(\dfrac{(\mu-T)^2}{\sigma^2}\right)\right]} = \frac{\chi^2_{n,\delta}}{n+\delta} \tag{3}$$

the relationship $\delta = n\xi^2 = n(\mu\text{-}T)^2/\sigma^2$ enables the distribution of $\hat{C}_{pp}$ to be determined as $C_{pp}\chi^2_{n,\delta}/n+\delta$. However, the dependency on ξ can be circumvented by adopting a conservative approach in determining the sample size (n), as proposed by Sheu et al. (2014), who suggested setting $\xi = 0$. This assumption corresponds to the process mean being centred at the target value (T), thereby ensuring a reliable sample size for decision-making. Under this condition, the cumulative distribution function (CDF) of can be derived as follows:

$$F_{C_{pp}}(y) = P(\hat{C}_{pp} \le y) = P\left(\chi^2_{n,n\left(1+\xi^2\right)} \le \frac{n\left(1+\xi^2\right)y}{C_{pp}}\right). \tag{4}$$

The process incapability index has undergone significant evolution since its inception, with numerous researchers contributing to its development and refinement. Building on the foundational work of Chen (1998) and Pearn & Lin (2001), subsequent studies have expanded its applicability and effectiveness in assessing process performance. Notable contributions include Wu & Yang (2002), Pearn et al. (2002), Chen et al. (2005), and Lin (2007). Recent investigations, such as those by Kahraman & Kaya (2011), Kaya & Baracli (2012), and Kaya (2014), have further broadened the index's utility. The development of variable single sampling plans based on the process incapability index, proposed by Sheu et al. (2014), marks a significant milestone in its application. Further advancements in the process incapability index have been made by Liao (2015), Gildeh & Ganji (2016), Ganji (2019), Gildeh & Ganji (2020), Bidabadi et al. (2021), Leony & Lin (2022), Pakzad & Basiri (2023), Chen et al. (2024), and Bera & Anis (2024), who have refined and enhanced its utility in assessing process performance. These contributions underscore the index's evolving nature and the ongoing efforts to improve its effectiveness. The review study by Yum (2023) highlights the importance of ongoing research in this area.

3 METHODOLOGY

The VQSS framework employed in this study comprises two distinct single-sampling plans, one for normal inspection and one for tightened inspection, with clearly defined rules governing the switching between these plans. Two primary variants of the VQSS framework are explored: VQSS(n_N, n_T; k) and VQSS (n_N, $n_T = mn_N$; k). The first variant is characterised by distinct sample sizes for normal and tightened inspections, denoted as n_N and n_T, respectively, with a common critical threshold k. In contrast, the second variant assumes a proportional relationship between the sample sizes, where the tightened inspection sample size (n_T) is determined as a multiple (m) of the normal inspection sample size (n_N).

3.1 VQSS(n_N, n_T; k)

The implementation of VQSS(n_N, n_T; k) based on the estimated Process Incapability Index ($\hat{C}_{pp}$) involves a series of steps, as depicted in Figure 1. Under the assumption of a normal distribution with two-sided specification limits, the process can be operationalised as follows:

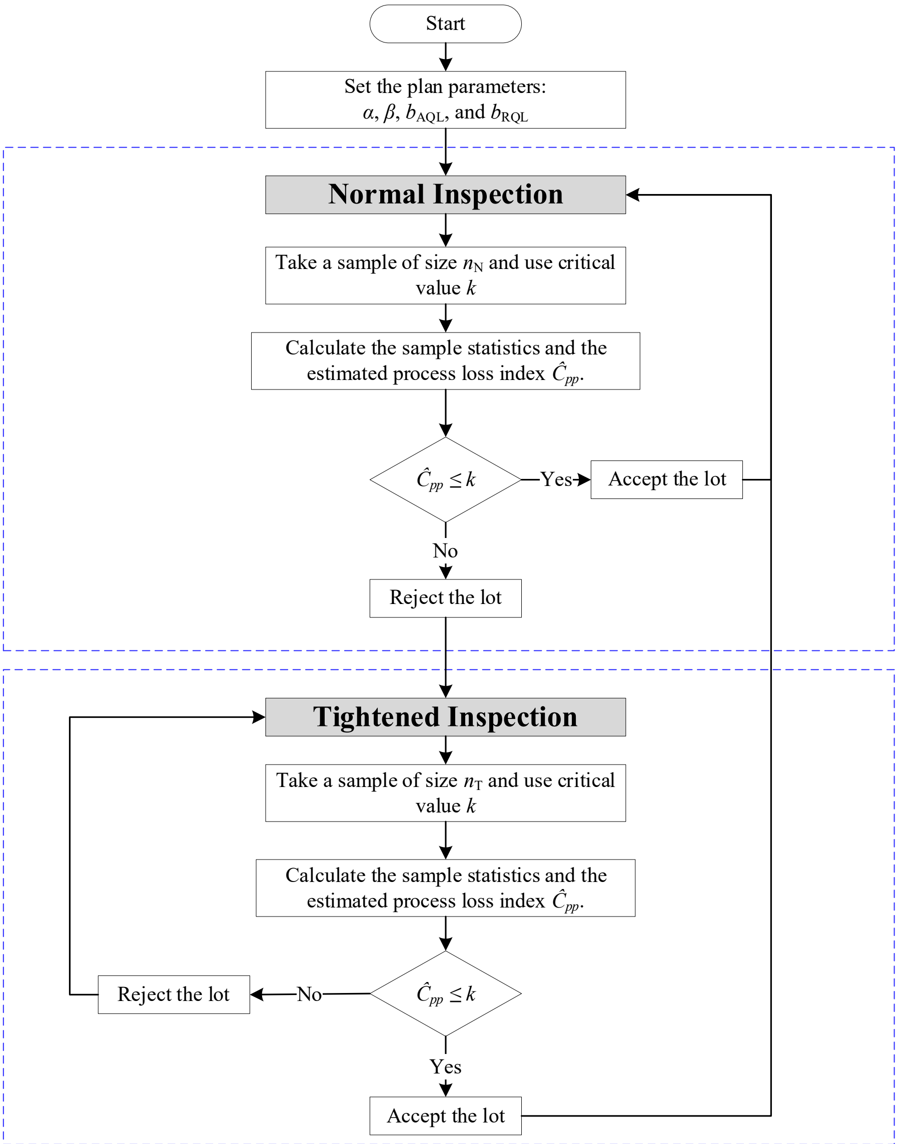


Figure 1 – Flow chart of VQSS(n_N, n_T; k)

Define the producer's risk (α) and the consumer's risk (β). Establish the acceptable quality level (b_{AQL}) and the unacceptable quality level (b_{RQL}). Establish the critical value k using the sampling distribution of the calculated process incapability index ($\hat{C}_{pp}$), along with the sample sizes (n_N and n_T). Based on the flowchart in Figure 1, the VQSS(n_N, n_T; k) procedure is operationalised through a two-stage inspection process. Firstly, under normal inspection, a random sample of size n_N is drawn from the lot, and the process incapability index ($\hat{C}_{pp}$) is estimated based on the sample statistics. The lot is accepted if $\hat{C}_{pp} \le k$. Otherwise, it is rejected, and the inspection mode is switched to Tightened Inspection. Secondly, in the tightened inspection mode, a new sample of size n_{T} is drawn, and $\hat{C}_{pp}$ is recalculated. If $\hat{C}_{pp} \le k$, the lot is accepted, and the inspection mode reverts to normal Inspection for the next lot. However, if $\hat{C}_{pp} > k$, the lot is rejected, and the inspection mode remains in tightened Inspection for the next lot or production is halted if quality deterioration persists.

By following the operational procedure outlined earlier, the probability of lot acceptance $P^{\mathrm{C}}(C_{pp})$, also known as the Operating Characteristic (OC) function, for the VQSS(n_{N}, n_{T}; k) plan can be formulated in terms of the process incapability index $\hat{C}_{pp}$. This formulation provides a mathematical basis for evaluating the performance of the VQSS plan as expressed.

$$P_N^C(C_{pp}) = P\left(\chi^2_{n_{\mathrm{N}},n_{\mathrm{N}}\left(1+\xi^2\right)} \le \frac{n_{\mathrm{N}}\left(1+\xi^2\right)\times k}{C_{pp}}\right), \tag{5}$$

$$P_T^C(C_{pp}) = P\left(\chi^2_{n_{\mathrm{T}},n_{\mathrm{T}}\left(1+\xi^2\right)} \le \frac{n_{\mathrm{T}}\left(1+\xi^2\right)\times k}{C_{pp}}\right). \tag{6}$$

The OC function's performance is evaluated based on its ability to meet the producer's and consumer's risk requirements, which requires the OC function to meet the two critical points: (b_{AQL}, $1-\alpha$) and (b_{RQL}, β). By imposing these two-point conditions on the mathematical expression for the OC function (Eq. (5) and (6)), we can derive the necessary equations to ensure that the sampling plan meets the desired risk levels as formulated below:

$$\pi_A^{\mathrm{C}}(b_{\mathrm{AQL}}) = \frac{P_T^C(b_{\mathrm{AQL}})}{1-P_N^C(b_{\mathrm{AQL}})+P_T^C(b_{\mathrm{AQL}})}, \tag{7}$$

$$\pi_A^{\mathrm{C}}(b_{\mathrm{RQL}}) = \frac{P_T^C(b_{\mathrm{RQL}})}{1-P_N^{\mathrm{C}}(b_{\mathrm{RQL}})+P_T^{\mathrm{C}}(b_{\mathrm{RQL}})}, \tag{8}$$

Given that VQSS involves two inspection modes with varying sample sizes, the ASN is a more suitable performance metric. The ASN, which represents the expected number of units inspected before a decision is made, can be calculated as:

$$\mathrm{ASN}(C_{pp}) = \frac{P_T^C(C_{pp})\times n_N + [1-P_N^{\mathrm{C}}(C_{pp})]\times n_T}{1-P_N^{\mathrm{C}}(C_{pp})+P_T^{\mathrm{C}}(C_{pp})}. \tag{9}$$

The determination of plan parameters (n_{N}, n_{T}, k) necessitates the simultaneous solution of the two-point OC equations. However, due to the potential existence of multiple parameter sets that satisfy these constraints, the ASN is employed as the objective function to be minimised. This leads to the formulation of an optimisation model that seeks to identify the optimal combination of nN, nT, and k that not only minimises ASN but also adheres to the specified producer's and consumer's risk levels.

$$\underset{n_{\mathrm{N}},n_{\mathrm{T}},k}{\mathrm{Min}}\ \mathrm{ASN}(b_{\mathrm{AQL}}) = \frac{P_T^C(b_{\mathrm{AQL}})\times n_N + [1-P_N^C(b_{\mathrm{AQL}})]\times n_T}{1-P_N^C(b_{\mathrm{AQL}})+P_T^C(b_{\mathrm{AQL}})} \tag{10}$$

Subject to

$$\pi_A^{\mathrm{C}}(b_{\mathrm{AQL}}) \geq 1-\alpha\ ,$$

$$\pi_A^{\mathrm{C}}(b_{\mathrm{RQL}}) \leq \beta\ ,$$

$$n_{\mathrm{T}} > n_{\mathrm{N}} > 1, b_{\mathrm{AQL}} \leq k \leq b_{\mathrm{RQL}}\ .$$

3.2 VQSS(n_{N}, n_{T}=mn_{N}; k)

In order to streamline the parameter determination process and enhance practical implementation, a specialised form of VQSS(n_{N}, n_{T}; k) is proposed, where the tightened-inspection sample size n_{T} is assumed to be a multiple m of the normal-inspection sample size n_{N} (i.e., $n_{\mathrm{T}} = m\times n_{\mathrm{N}}$ with $m > 1$). This variant, denoted as VQSS(n_{N}, n_{T}=mn_{N}; k), retains the same operational logic, acceptance probability function, and mathematical formulation as the general VQSS model. By imposing this constraint, the complexity of the optimisation procedure is reduced, as only two parameters (n_{N} and k) need to be determined. Moreover, the VQSS(n_{N}, n_{T}=mn_{N}; k) model encompasses the conventional VSS plan (Sheu et al., 2014) as a special case when $m = 1$, thereby providing a generalised extension of the VSS plan and broadening its applicability.

4 RESULT ANALYSIS AND DISCUSSION

To determine the optimal plan parameters for the VQSS within the framework established in this study, the sequential quadratic programming (SQP) algorithm was employed, leveraging the "*fmincon*" function available in MATLAB R2019a. This approach facilitated the efficient solution of the optimisation problem, yielding the desired plan parameters.

4.1 Plan Parameters

4.1.1 VQSS (n_{N}, n_{T}, k)

To implement VQSS(n_{T}, n_{N}; k), three plan parameters need to be determined concurrently: the sample sizes for tightened inspection (n_{T}) and normal inspection (n_{N}), along with the critical value (k). Table 2 presents the plan parameters for VQSS(n_{N}, n_{T}; k) under various combinations that can be used by practitioners to carry out the two-plan sampling system. For example, if the producer and the consumer have predetermined the conditions of $(b_{AQL}, b_{RQL}) = (0.5917, 0.1000)$ and $(\alpha, \beta) = (0.05, 0.05)$, plan parameters $(n_N, n_T; k) = (32, 273, 0.8523)$ can be obtained from Table 2. This means that 32 samples should be taken under normal inspection and 273 samples should be taken under tightened inspection, with a critical value of 0.8523. Afterwards, the C_{pp} can be calculated to decide whether to accept or reject the inspected lot. If C_{pp} exceeds the critical value $k = 0.8523$, the lot will be rejected; otherwise, it will be accepted. Moreover, if the lot is rejected under normal inspection, the inspection system has to be switched to a tightened inspection. However, when the lot is accepted during tightened inspection, normal inspection must be carried out for the next submitted lot to ensure the quality of the delivered products.

Table 2 – Plan parameters of VQSS(n_N, n_T; k)

α	β	b_{AQL} = 0.5917, b_{RQL} = 1.000			b_{AQL} = 0.4444, b_{RQL} = 0.5917			b_{AQL} = 0.3673, b_{RQL} = 0.4444			b_{AQL} = 0.2500, b_{RQL} = 0.3673		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	50	1250	0.9038	180	4231	0.5605	420	9580	0.4288	97	2335	0.3413
	0.050	46	942	0.9171	168	3187	0.5650	392	7213	0.4310	90	1759	0.3450
	0.100	44	778	0.9254	161	2626	0.5678	375	5939	0.4325	86	1451	0.3473
0.050	0.010	36	389	0.8340	131	1293	0.5369	306	2907	0.4167	71	719	0.3220
	0.050	32	273	0.8523	115	906	0.5433	269	2037	0.4201	62	504	0.3272
	0.100	29	215	0.8644	106	711	0.5476	248	1597	0.4223	57	396	0.3306
0.100	0.010	29	230	0.7887	104	757	0.5210	242	1695	0.4086	56	423	0.3092
	0.050	24	155	0.8081	87	510	0.5282	204	1140	0.4124	47	285	0.3149
	0.100	21	120	0.8216	78	391	0.5332	182	874	0.4150	42	219	0.3188

4.1.2 VQSS(n_N, n_T=mn_N; k)

To simplify the determination of necessary parameters and implementation of VQSS in practical scenarios, an alternative type of VQSS(n_N, n_T; k) is proposed. This type assumes that the sample size for tightened inspection (n_T) is equal to m times of the sample size for normal inspection (n_N), represented as $m\times n_N$, where $m > 1$. It is important to note that when $m = 1$, VQSS and VSSP are identical. The solved plan parameters for the VQSS(n_N, n_T=mn_N; k) under various conditions are provided in Tables 2-5, with m values of 1.5, 2.0, 2.5, and 3.0. These results offer practical guidance for practitioners and simplify the implementation of VQSS in their inspection processes.

Table 2 – Plan parameters of VQSS(n_N, n_T; k) under m = 1.5

α	β	b_{AQL} = 0.5917, b_{RQL} = 1.000			b_{AQL} = 0.4444, b_{RQL} = 0.5917			b_{AQL} = 0.3673, b_{RQL} = 0.4444			b_{AQL} = 0.2500, b_{RQL} = 0.3673		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	128	192	0.7772	432	648	0.5177	978	1467	0.4070	239	358	0.3063

α	β	b_{AQL} = 0.5917, b_{RQL} = 1.000			b_{AQL} = 0.4444, b_{RQL} = 0.5917			b_{AQL} = 0.3673, b_{RQL} = 0.4444			b_{AQL} = 0.2500, b_{RQL} = 0.3673		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
	0.050	95	142	0.8104	323	484	0.5298	734	1100	0.4133	177	266	0.3159
	0.100	79	118	0.8333	271	406	0.5380	618	927	0.4176	148	222	0.3224
0.050	0.010	93	139	0.7407	307	461	0.5045	691	1037	0.4001	171	256	0.2958
	0.050	64	96	0.7722	216	324	0.5164	489	733	0.4065	120	179	0.3051
	0.100	51	77	0.7949	174	261	0.5249	395	593	0.4109	96	144	0.3118
0.100	0.010	76	113	0.7158	249	373	0.4952	557	836	0.3952	139	208	0.2884
	0.050	50	75	0.7445	168	251	0.5065	377	566	0.4013	93	139	0.2972
	0.100	39	59	0.7659	131	196	0.5148	296	444	0.4058	73	109	0.3036

Table 3 – Plan parameters of VQSS(n_N, n_T; k) under m = 2.0

α	β	b_{AQL} = 0.5917, b_{RQL} = 1.000			b_{AQL} = 0.4444, b_{RQL} = 0.5917			b_{AQL} = 0.3673, b_{RQL} = 0.4444			b_{AQL} = 0.2500, b_{RQL} = 0.3673		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	112	223	0.7914	379	758	0.5228	861	1721	0.4097	209	417	0.3104
	0.050	85	169	0.8240	291	581	0.5346	663	1326	0.4158	159	318	0.3197
	0.100	72	143	0.8459	249	497	0.5423	569	1137	0.4198	136	271	0.3260
0.050	0.010	78	156	0.7538	262	523	0.5094	591	1182	0.4027	145	290	0.2996
	0.050	56	111	0.7853	189	378	0.5212	430	859	0.4090	104	208	0.3089
	0.100	46	91	0.8076	156	311	0.5294	354	708	0.4133	86	171	0.3154
0.100	0.010	63	125	0.7269	207	413	0.4996	465	930	0.3976	115	230	0.2918
	0.050	43	85	0.7561	143	286	0.5109	323	646	0.4037	79	158	0.3006
	0.100	34	67	0.7774	114	228	0.5191	259	517	0.4080	63	125	0.3070

For instance, if a contract specifies conditions such as (b_{AQL},b_{RQL})= (0.2500, 0.3673), (α, β) = (0.05, 0.10), and m = 1.5, then plan parameters (n_N, n_T; k) = (86, 171, 0.3154) can be obtained by referring to Table 2. This indicates that 86 samples must be collected under normal inspection, whereas 171 samples must be taken under tightened inspection. Then, $\hat{C}_{pp}$ can be calculated based on the collected samples and compared with the critical value for lot sentencing. If $\hat{C}_{pp} \leq 0.3154$, the lot is rejected; otherwise, if $\hat{C}_{pp} > 0.3154$, the lot is accepted.

Table 4 – Plan parameters of VQSS(n_N, n_T; k) under m = 2.5

α	β	b_{AQL} = 0.5917, b_{RQL} = 1.000			b_{AQL} = 0.4444, b_{RQL} = 0.5917			b_{AQL} = 0.3673, b_{RQL} = 0.4444			b_{AQL} = 0.2500, b_{RQL} = 0.3673		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	101	252	0.8025	345	862	0.5268	785	1962	0.4117	190	474	0.3135
	0.050	78	195	0.8343	270	674	0.5381	617	1542	0.4176	148	368	0.3226
	0.100	67	167	0.8554	234	584	0.5455	536	1339	0.4214	127	318	0.3286

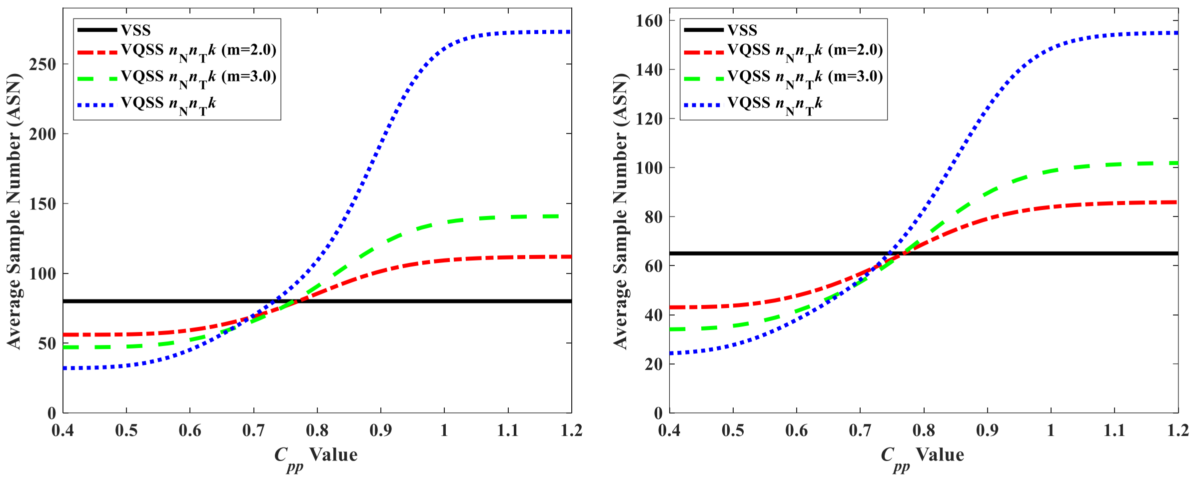
α	β	b_{AQL} = 0.5917, b_{RQL} = 1.000			b_{AQL} = 0.4444, b_{RQL} = 0.5917			b_{AQL} = 0.3673, b_{RQL} = 0.4444			b_{AQL} = 0.2500, b_{RQL} = 0.3673		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.050	0.010	69	173	0.7642	233	582	0.5133	528	1319	0.4047	129	322	0.3026
	0.050	51	126	0.7956	172	430	0.5249	392	979	0.4109	95	236	0.3119
	0.100	42	104	0.8174	144	358	0.5329	328	819	0.4151	79	196	0.3182
0.100	0.010	54	135	0.7360	181	451	0.5031	408	1018	0.3994	100	250	0.2945
	0.050	38	94	0.7654	128	318	0.5144	289	722	0.4055	70	175	0.3033
	0.100	30	75	0.7865	103	257	0.5225	235	586	0.4098	57	141	0.3096

Table 5 – Plan parameters of VQSS(n_N, n_T; k) under m = 3.0

α	β	b_{AQL} = 0.5917, b_{RQL} = 1.000			b_{AQL} = 0.4444, b_{RQL} = 0.5917			b_{AQL} = 0.3673, b_{RQL} = 0.4444			b_{AQL} = 0.2500, b_{RQL} = 0.3673		
		n_N	n_T	k	n_N	n_T	k	n_N	n_T	k	n_N	n_T	k
0.010	0.010	94	280	0.8114	321	962	0.5300	732	2194	0.4134	176	528	0.3161
	0.050	73	219	0.8426	255	764	0.5410	584	1751	0.4191	139	417	0.3249
	0.100	64	190	0.8630	223	668	0.5481	512	1536	0.4227	121	363	0.3307
0.050	0.010	63	188	0.7728	213	639	0.5164	484	1450	0.4064	118	352	0.3052
	0.050	47	139	0.8041	160	479	0.5280	365	1094	0.4125	88	262	0.3143
	0.100	39	116	0.8255	135	404	0.5357	309	925	0.4165	74	220	0.3205
0.100	0.010	49	145	0.7436	163	487	0.5060	368	1102	0.4010	90	270	0.2968
	0.050	34	102	0.7732	117	349	0.5173	265	795	0.4070	64	192	0.3056
	0.100	28	83	0.7941	95	285	0.5252	218	652	0.4112	52	156	0.3118

4.2 Average Sample Number (ASN)

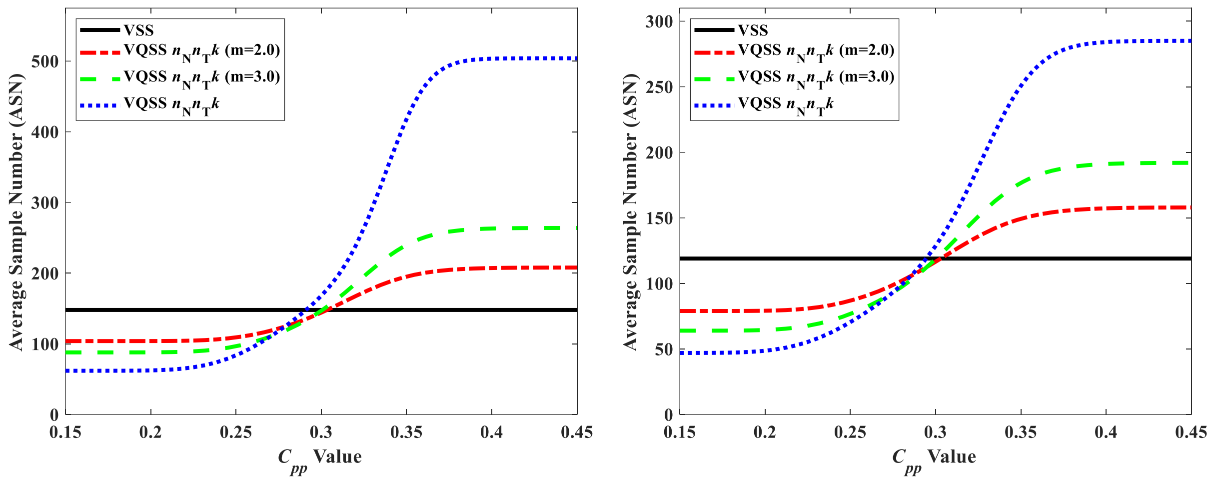
In this part, we evaluate and contrast the efficacy of the suggested VQSS and VSSP by analysing the OC and ASN curves. We first perform a performance evaluation of the two kinds of VQSS and VSSP through the OC curve.



(a) $(\alpha, \beta) = (0.05, 0.05)$ (b) $(\alpha, \beta) = (0.10, 0.05)$

Figure 2 – The ASN curves of the VSS, VQSS(n_N, n_T=mn_N; k) with m = 2.0, 3.0, VQSS(n_N, n_T; k) under (b_{AQL}, b_{RQL}) = (0.5917, 1.000)

As the primary overall measurement system, we create the ASN curves for VSSP and the two varieties of VQSS to assess their sampling effectiveness from a financial perspective. Figures 2-3 show the ASN curves for VSSP, VQSS(n_N, n_T; k), VQSS(n_N, n_T=mn_N; k) with varying m values (m = 2, 3), and risk levels of (α, β)=(0.05, 0.05), (0.10, 0.05), and quality levels (b_{AQL}, b_{RQL}) = (0.5917, 1.000) and (0.2500, 0.3673). It is clear that VSSP and VQSS(n_N, n_T=mn_N; k), with varying m values, and VQSS(n_N, n_T; k) significantly rely on the quality of the lot. When the lot is of high quality, both VQSS(n_N, n_T=mn_N; k) (independently of m's value) and VQSS(n_N, n_T; k) need a smaller sample size compared to VSSP. Furthermore, when the quality of the lot is inadequate (with a comparatively high value of $\hat{C}_{pp}$), VQSS(n_N, n_T=mn_N; k) and VQSS(n_N, n_T; k) typically need a greater quantity of sample items for evaluation, as tightened inspection may be essential to guarantee the quality of the lot.



(a) $(\alpha, \beta) = (0.05, 0.05)$ (b) $(\alpha, \beta) = (0.10, 0.05)$

Figure 3 – The ASN curves of the VSS, VQSS(n_N, $n_T=mn_N$; k) with m = 2.0, 3.0, VQSS(n_N, n_T; k) under (b_{AQL}, b_{RQL}) = (0.2500, 0.3673)

4.3 Operating Characteristics (OC) Curve

The OC curve is a graphical representation of a sampling plan's ability to discriminate between different quality levels, with the acceptance probability plotted against the quality level. The slope of the OC curve serves as an indicator of the plan's discriminatory power, with steeper slopes indicating better performance. The proposed VQSS system, which integrates Normal-VSS and Tightened-VSS plans, is subjected to further investigation in terms of its OC curve behaviour. This figure combines two models of sampling plans under combination (b_{AQL}, b_{RQL}) = (0.5917, 1.000), $(\alpha, \beta) = (0.05, 0.10)$ and m = 3: the Normal-VSS plan with (n_N, k) = (39, 0.8255) and the Tightened-VSS plan with (n_T, k) = (116, 0.8255). The VQSS system's performance is evaluated in comparison to its constituent plans, as illustrated in Figure 4. If the quality of the submitted lot is inadequate (i.e., it exceeds the critical value $k = 0.8255$), the OC curve of the VQSS system closely aligns with that of the Tightened-VSS plan. On the other hand, as the quality of the lot improves, the OC curve of the VQSS system approaches the curve of the Normal-VSS plan. This illustrates the VQSS system's adaptability in choosing the suitable inspection according to the real quality level, while preserving its discriminatory capability. The VQSS system adjusts to fluctuating quality standards, rendering it a reliable and efficient sampling solution. Utilising this flexibility, the VQSS system can offer efficient quality control while reducing unnecessary inspections.

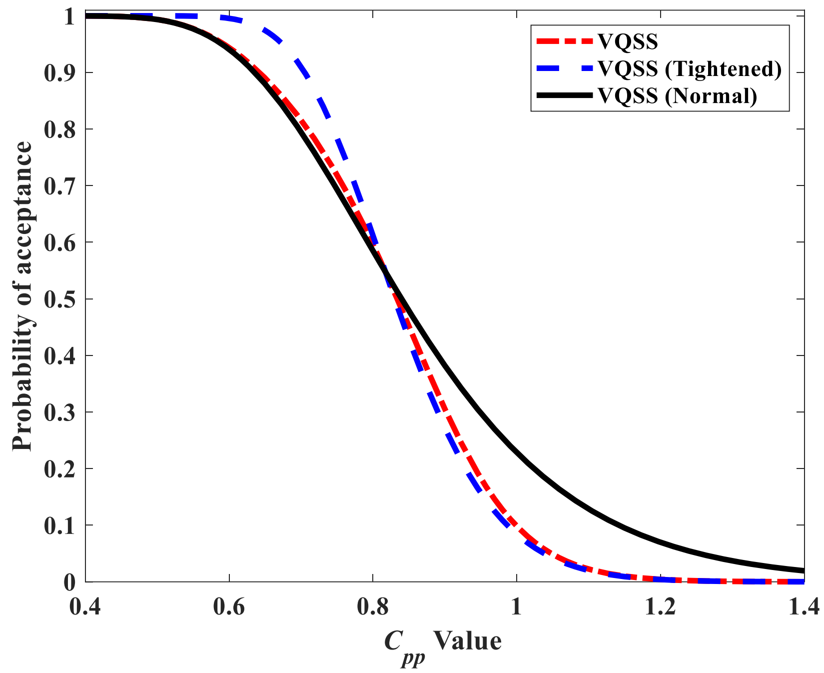
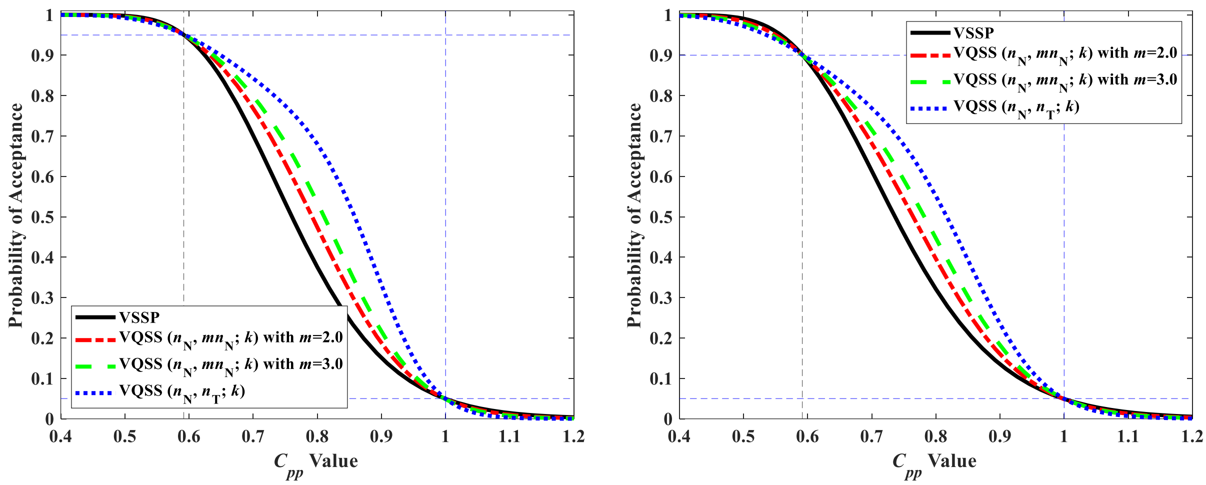


Figure 4 – The OC curves of Normal-VSS plans, Tightened-VSS plans, and the VQSS system.

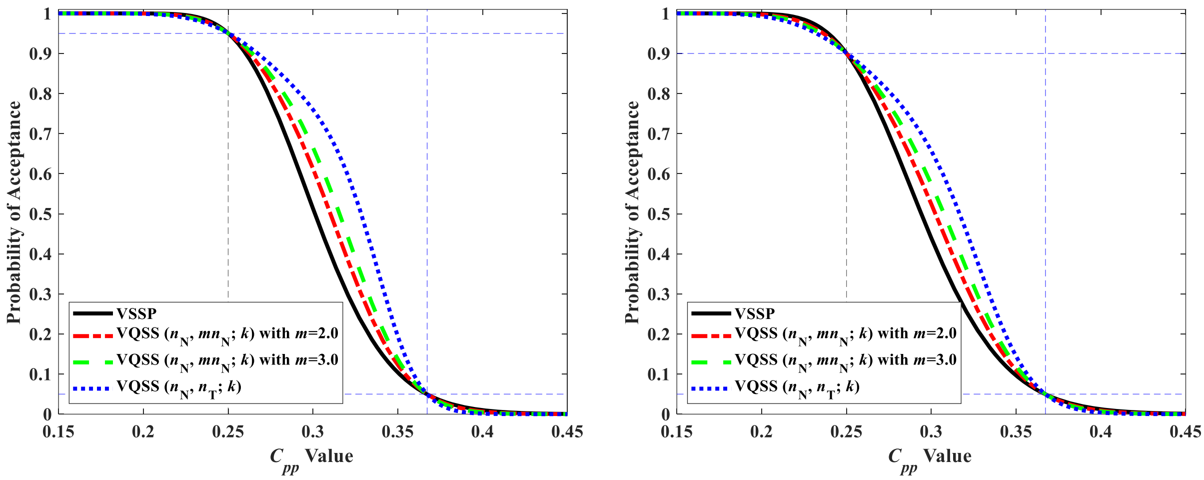


(a) $(\alpha, \beta) = (0.05, 0.05)$ (b) $(\alpha, \beta) = (0.10, 0.05)$

Figure 5 – The OC curves of the VSSP, VQSS(n_N, n_T=mn_N; k) with m = 2.0, 3.0, VQSS(n_N, n_T; k), and (b_{AQL}, b_{RQL}) = (0.5917, 1.000).

Figures 5 and 6 present the OC curves for the Variable Single Sampling (VSS) plan and the Variable Quick Switching Sampling (VQSS) system. Both plans are based on the process incapability index $\hat{C}_{pp}$. They are evaluated under specific conditions, including two sets of AQL and RQL values (b_{AQL}, b_{RQL}) = (0.5917, 1.000) and (0.2500, 0.3673), along with their corresponding risk levels (α, β) =

(0.05, 0.05) and (α, β) = (0.10, 0.05). The results demonstrate that both OC curves conform to the required conditions, passing through the designated points (b_{AQL}, 1 - α) and (b_{RQL}, β).



(a) $(\alpha, \beta) = (0.05, 0.05)$ (b) $(\alpha, \beta) = (0.10, 0.05)$

Figure 6 – The OC curves of the VSSP, VQSS(n_N, n_T=mn_N; k) with m = 2.0, 3.0, VQSS(n_N, n_T; k), and (b_{AQL}, b_{RQL}) = (0.2500, 0.3673) .

Significantly, the OC curve of the VQSS system displays a moderate form compared to the ideal OC curve, indicating its ability to differentiate between acceptable and unacceptable quality levels. The data show that for the VQSS system with parameters VQSS(n_{N}, n_{T} = mn_{N}; k), a higher value of m results in a steeper slope of the OC curve, reflecting enhanced discriminatory ability. In other terms, greater values of m improve the system's capacity to differentiate between good- and poor-quality lots.

5 CONCLUSION

In the current competitive market environment, companies must focus on product quality to meet the growing demands of their clientele. Conventional acceptance sampling methods typically assess product quality by looking at process yield, which overlooks minor variations within specification limits. To address this limitation, the process incapability index (C_{pp}) was developed to measure process performance, taking into account both process accuracy and precision. This research presents an innovative two-stage sampling method, VQSS, which utilises the Cpp index to modify inspection rigour in response to variations in product quality. Through the integration of both tightened and normal inspection protocols, VQSS provides improved adaptability relative to traditional single sampling (VSS) plans. Two versions of VQSS were created and thoroughly assessed by utilising operating characteristic (OC) and average sample number (ASN) curves. The findings indicate that VQSS outperforms VSSP in terms of adaptability and efficiency. By implementing VQSS, organisations can enhance their quality control procedures and respond more efficiently to fluctuations in product quality. This research contributes to the advancement of quality control techniques and provides valuable insights for sectors seeking to enhance their product quality.

The proposed model, VQSS(n_N, n_T; k) and VQSS(n_N, $n_T = mn_N$; k), offers a benefit in terms of sample size modification when transitioning to a stricter inspection plan due to a decline in lot quality. Moreover, these VQSS variants provide additional details for evaluating lot quality, motivating suppliers to improve product quality and reduce possible expenses. To aid implementation, the research provides detailed tables of plan parameters for every VQSS type, catering to various quality scenarios and risk combinations. Industries implementing VQSS can enhance their quality control systems and adjust to evolving product quality standards. The proposed system provides a beneficial framework for quality control professionals seeking to improve their inspection procedures.

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REFERENCES

Balamurali, S. and Usha, M. 2012. Variables quick switching system with double specification limits, *International Journal of Reliability, Quality and Safety Engineering*, 19(2), pp. 1–17. https://doi.org/10.1142/S0218539312500088

Balamurali, S. and Usha, M. 2014. Optimal designing of variables quick switching system based on the process capability index Cpk, *Journal of Industrial and Production Engineering*, 31(2), pp. 85–94. https://doi.org/10.1080/21681015.2014.893929

Balamurali, S. and Usha, M. 2017. Developing and designing of an efficient variables sampling system based on the process capability index, *Journal of Statistical Computation and Simulation*, 87(7), pp. 1401–1415. https://doi.org/10.1080/00949655.2016.1267735

Bera, K. and Anis, M. Z. 2024. Process incapability index for autocorrelated data in the presence of measurement errors, *Communications in Statistics - Theory and Methods*, 53(15), pp. 5439–5459. https://doi.org/10.1080/03610926.2023.2220921

Bidabadi, H. S., Shishebori, D. and Yazdi, A. A. 2021. Multivariate Process Incapability Index Considering Measurement Error in Fuzzy Environment, *Advances in Industrial Engineering*, 54(2), pp. 205–220. https://doi.org/10.22059/jieng.2021.323883.1765

Chen, K. et al. 2024. Fuzzy Testing Method of Process Incapability Index, *Mathematics*, 12(5), p. 623. https://doi.org/10.3390/math12050623

Chen, K. S. 1998. Estimation of the process incapability index, *Communications in Statistics - Theory and Methods*, 27(5), pp. 1263–1274. https://doi.org/10.1080/03610929808832157

Chen, K. S. 1998. Incapability index with asymmetric tolerances, *Statistica Sinica*, 8(1), pp. 253–262. http://www.jstor.org/stable/24306353

Chen, K. S., Chen, K. L. and Li, R. K. 2005. Contract manufacturer selection by using the process incapability index C-pp, *International Journal of Advanced Manufacturing Technology*, 26(5–6), pp. 686–692. https://doi.org/10.1007/s00170-003-1886-5

Chen, K. S. and Chen, T. W. 2008. Multi-process capability plot and fuzzy inference evaluation, *International Journal of Production Economics*, 111(1), pp. 70–79. https://doi.org/10.1016/j.ijpe.2006.12.056

Darmawan, A., Bahri, S., et al. 2025. A flexible resubmitted variable sampling plan for product quality determination using the process loss index, *Production Engineering Archives*, 31(2), pp. 201–211. https://doi.org/10.30657/pea.2025.31.20

Darmawan, A., Wu, C., et al. 2025. Developing variables two-plan sampling scheme with consideration of process loss for lot sentencing, *Quality Engineering*, 37(2), pp. 273–291. https://doi.org/10.1080/08982112.2024.2381012

Dodge, H. F. 1965. Evaluation of Sampling Inspection System having Rules for Switching between Normal and Tightened Inspection, In: Technical Report 14. Rutgers State University: The Statistics Center.

Ganji, Z. A. 2019. Multivariate process incapability vector, *Quality and Reliability Engineering International*, 35(4), pp. 902–919. https://doi.org/10.1002/qre.2435

Ganji, Z. A. and Gildeh, B. S. 2016. Assessing Process Performance with Incapability Index Based on Fuzzy Critical Value, *Iranian Journal of Fuzzy Systems*, 13(5), pp. 21–34. https://doi.org/10.22111/ijfs.2016.2731

Gildeh, B. S. and Ganji, Z. A. 2020. The effect of measurement error on the process incapability index, *Communications in Statistics-Theory and Methods*, 49(3), pp. 552–566. https://doi.org/10.1080/03610926.2018.1543777

Govindaraju, K. and Ganesalingam, S. 1998. Zero acceptance number quick switching system for compliance sampling, *Journal of Applied Statistics*, 25(3), pp. 399–407. https://doi.org/10.1080/02664769823124

Greenwich M, and Jahr-Schaffrath B.L. 1995. A process incapability index. *International Journal of Quality & Reliability Management*, 12(4), pp. 58–71. https://doi.org/10.1108/02656719510087328

Hald, A. and Thyregod, P. 1965. The composite operating characteristic under normal and tightened sampling inspection by attributes, *Bulletin of International Statistical Institute*, 41, pp. 517–529.

Kahraman, C. and Kaya, I. 2011. Fuzzy Estimations of Process Incapability Index, World Congress on Engineering, WCE 2011, VOL II. Edited by S. I. Ao et al. Istanbul Technical University, Department of Industrial Engineering, TR-34367 Macka, Istanbul, Turkey, pp. 1106–1110.

Kaya, I. 2014. The Process Incapability Index under Fuzziness with an Application for Decision Making, *International Journal of Computational Intelligence Systems*, 7(1), pp. 114–128. doi: https://doi.org/10.1080/18756891.2013.858905.

Kaya, I. and Baracli, H. 2012. Fuzzy Process Incapability Index with Asymmetric Tolerances, *Journal of Multiple-Valued Logic and Soft Computing*, 18(5–6), pp. 493–511.

Leony, F. and Lin, C. 2022. The PO bootstrap approach for comparing process incapability applied to non-normal process selection, *Quality Technology & Quantitative Management*, 19(2), pp. 215–233. https://doi.org/10.1080/16843703.2021.2015827

Liao, M. Y. 2015. Assessing process incapability when collecting data from multiple batches, *International Journal of Production Research*, 53(7), pp. 2041–2054. https://doi.org/10.1080/00207543.2014.952796

Lin, G. H. 2007. A Bayesian approach based on multiple samples for measuring process performance with incapability index, *International Journal of Production Economics*, 106(2), pp. 506–512. https://doi.org/10.1016/j.ijpe.2006.06.012

Liu, S. W., Wang, Z. H. and Wang, T. C. 2023. Developing a cost-efficient dual sampling system for lot disposition by considering process yield and quality loss, *Quality Engineering*, 35(2), pp. 267–278. https://doi.org/10.1080/08982112.2022.2124381

Liu, S. W. and Wu, C. W. 2016. A quick switching sampling system by variables for controlling lot fraction nonconforming, *International Journal of Production Research*, 54(6), pp. 1839–1849. https://doi.org/10.1080/00207543.2015.1084062

Montgomery, D. C. 2019. *Introduction to Statistical Quality Control*. 8th edn, John Wiley & Sons, Inc. 8th edn. Edited by S. Dumas. NJ: John Wiley & Sons, Inc.

Pakzad, A. and Basiri, E. 2023. A new incapability index for simple linear profile with asymmetric tolerances, *Quality Engineering*, 35(2), pp. 324–340. https://doi.org/10.1080/08982112.2022.2129025

Pearn, W. L., Chen, K. L. and Chen, K. S. 2002. A practical implementation of the incapability index C-pp, *International Journal of Industrial Engineering-Theory Applications and Practice*, 9(4), pp. 372–383.

Pearn, W. L. and Lin, G. H. 2001. On the reliability of the estimated incapability index, *Quality and Reliability Engineering International*, 17(4), pp. 279–290. https://doi.org/10.1002/qre.378

Romboski, L. D. 1969. *An Investigation of Quick Switching Acceptance Sampling Systems*, Ph.D. Thesis, Rutgers-The State University.

Schilling, E. G. 1982. *Acceptance Sampling in Quality Control*, New York. Edited by D. B. Owen. New York: Marcel Dekker, Inc.

Sheu, L. C. et al. 2014. Developing acceptance sampling plans based on incapability index Cpp, *Applied Mathematics and Information Sciences*, 8(5), pp. 2509–2514. https://doi.org/10.12785/amis/080548

Soundararajan, V. and Arumainayagam, S. D. 1990. Construction and selection of modified quick switching systems, *Journal of Applied Statistics*, 17(1), pp. 83–114. https://doi.org/10.1080/757582650

Soundararajan, V. and Palanivel, M. 2000. Quick switching variables single sampling (QSVSS) system indexed by AQL and AOQL, *Journal of Applied Statistics*, 27(6), pp. 771–778. https://doi.org/10.1080/02664760050081942

Wang, T.-C. 2022. Developing an adaptive sampling system indexed by Taguchi capability with acceptance-criterion-switching mechanism, *The International Journal of Advanced Manufacturing Technology*, 122, pp. 2329–2342. https://doi.org/10.1007/s00170-022-09996-2

Wang, T. C. and Shu, M. H. 2023. Development of an adaptive sampling system based on a process capability index with flexible switching mechanism, *International Journal of Production Research*, 61(21), pp. 7233–7247. https://doi.org/10.1080/00207543.2022.2147236

Wang, T. C. and Wu, C. W. 2025. Adaptive sampling system with quick-switching mechanism indexed by lifetime performance for product reliability verification, *Annals of Operations Research*. https://doi.org/10.1007/s10479-025-06555-2

Wang, T. C., Wu, C. W. and Wang, H. Y. 2025. Developing generalized quick-switch sampling systems for high-yield product verification, . https://doi.org/10.1016/j.cie.2025.111202

Wang, Z., Wu, C.-W. and Jhu, J. J. 2021. Design and construction of a quick-switching sampling system with a third-generation capability index, *Communications in Statistics - Theory and Methods*, 52(11), pp. 3633–3651. https://doi.org/10.1080/03610926.2021.1977959

Wu, C. W. et al. 2017. Capability-based quick switching sampling system for lot disposition, *Applied Mathematical Modelling*, 52, pp. 131–144. https://doi.org/10.1016/j.apm.2017.07.050

Wu, C. W. et al. 2024. Integrating capability index and generalized rule-switching mechanism for enhanced quick-switch sampling systems, *International Journal of Production Economics*, 276. https://doi.org/10.1016/j.ijpe.2024.109366

Wu, C. W., and Darmawan, A. 2025. A modified sampling scheme for lot sentencing based on the third-generation capability index. *Annals of Operations Research,* 349, pp. 25–46. https://doi.org/10.1007/s10479-023-05328-z

Wu, J. U. and Yang, C. C. 2002. Estimated incapability index: Reliability and decision making with sample information, *Quality and Reliability Engineering International*, 18(2), pp. 141–147. https://doi.org/10.1002/qre.455

Yum, B. J. 2023. A bibliography of the literature on process capability indices (PCIs): 2010–2021, Part I: Books, review/overview papers, and univariate PCI-related papers, *Quality and Reliability Engineering International*, 39(4), pp. 1413–1438. https://doi.org/10.1002/qre.3258

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CONFLICTS OF INTEREST

The authors declare no conflict of interest. The funders had no role in the design of the study, in the collection, analyses, or interpretation of data, in the writing of the manuscript, or in the decision to publish the results.

DISCLOSURE OF ARTIFICIAL INTELLIGENCE ASSISTANCE

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