Estimating the Process Sigma Level Based on Information from Statistical Process Control or Process Capability Analysis

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ABSTRACT

Purpose: The article proposes an approach to estimating the process sigma level based on information from statistical process control or process capability analysis.

Methodology/Approach: The *z*-score for each critical-to-quality characteristic and for the overall process is estimated from the data obtained in statistical process control or process capability analysis.

Findings: The proposed procedure for estimating the sigma level offers several advantages over the traditional method.

Research Limitation/Implication: We assume that each critical-to-quality characteristic is linked to only one type of defect opportunity, which can result in only one defect of that type on a single unit. We only consider continuous, normally distributed, and nominal critical-to-quality characteristics with two possible values: non-defective and defective.

Originality/Value of paper: The proposed procedure enables us to use information from statistical process control and process capability analysis to estimate the process sigma level and provides additional valuable insights into the process.

Category: Research paper

Keywords: process sigma level; critical-to-quality characteristic; defect; defect opportunity

Research Areas: Quality Engineering

1 INTRODUCTION

The article proposes an alternative method for estimating the process sigma level (z-score) that offers several benefits over the traditional procedure.

If the process is measurably incapable and variability is the dominant problem, statistical process control (SPC) is typically applied to understand and stabilise variability. Once the process is stabilised, the process capability is estimated, often using process capability indices. Utilising both SPC and capability indices requires estimating the probability distribution parameters of the quality characteristics. If SPC indicates that routine control measures are inadequate or if the process capability indices are low, it may prompt consideration of a Six Sigma project. These are just two potential reasons to initiate a Six Sigma project within an organisation; there can be other specific quantitative and qualitative factors that lead to this decision.

The Six Sigma approach is project-oriented and focused on the organisation's strategic business goals. The primary purpose of the Six Sigma project is to address a specific problem and contribute to achieving the organisation's business objectives. The fundamental philosophy is centred around increasing customer satisfaction by eliminating and preventing nonconformities, ultimately leading to increased business profitability (Zgodavová et al., 2020). The primary advantage of this methodology is its rigorous approach to identifying and eliminating sources of variability (Stamatis, 2003, p. 9). Six Sigma activities are associated with the DMAIC procedure, which comprises five phases: Define, Measure, Analyse, Improve, and Control. In the define phase, the problem to be addressed is identified and clearly defined. The measure phase focuses on measuring the current performance of the process to be improved. In the analysis phase, the main causes of poor performance are identified. The improvement phase aims to test and evaluate potential solutions to create a robust, improved process. In the control phase, improved processes are monitored through standardized procedures that can be operated and continuously refined, ensuring that performance improvements are sustained over time. The DMAIC is widely used to reduce defects, minimize variation, and enhance process capability.

Six Sigma indicators are measurable metrics utilized to evaluate the performance and progress of Six Sigma projects and processes. Key metrics include Defects per Opportunity (DPO), Defects Per Million Opportunities (DPMO), and Defects per Unit (DPU) (Costa et al., 2019). The first step in initiating a Six Sigma project is to estimate the process sigma level using the DPMO metric. Traditionally, DPMO is estimated from a random sample of products, and the result is then used to determine the process's z-score.

When implementing statistical process control, it is essential to estimate the means and standard deviations of continuous characteristics. During capability analysis, these parameters must also be estimated when calculating the C_{pk} capability index. For nominal characteristics, the proportion of defective units is estimated, instead of these parameters. Once the estimates of these parameters are available, they can also be used to calculate DPMO and the z-scores for individual characteristics, as well as the overall z-score for the product manufacturing process. Therefore, selecting a new sample is unnecessary.

We will begin with the method for calculating the DPMO characteristic and the capability indices C_p and C_{pk}. We will show how to calculate DPMO and the corresponding sigma level for individual characteristics and for the entire process. This will use the estimated distribution parameters of the critical-to-quality characteristics obtained during the implementation of statistical process control or process capability analysis. To clarify this calculation procedure, we will illustrate it with an example.

2 LITERATURE REVIEW

When assessing a single product, multiple critical-to-quality characteristics are typically monitored. The sigma level (z-score) for different critical-to-quality characteristics is determined based on the DPMO indicator value. Its value is usually estimated from a random sample by the value of the sample $\widehat{\text{DPMO}}$, calculated as follows (ISO 13053 – 1 (2011):

$$\widehat{\text{DPMO}} = \frac{c}{n_{\text{units}} \cdot n_{\text{CTQC}}} \cdot 1{,}000{,}000 = \widehat{\text{DPO}} \cdot 1{,}000{,}000 \tag{1}$$

where $\widehat{\text{DPMO}}$ is the sample number of defects per million opportunities,

c is the number of defects in the sample,

n_{units} is the sample size,

n_{CTQC} is the number of critical-to-quality characteristics,

$n_{\text{units}} \cdot n_{\text{CTQC}}$ are the number of defect opportunities in the sample,

$\widehat{\text{DPO}}$ is the sample number of defects per opportunity.

Based on this value, the process sigma level (z-score) is determined (see, for example, ISO 13053-1, 2011; Terek, 2023).

Six Sigma is closely linked with process capability analysis. One of the process capability analysis tools is the capability indices. Let us consider a characteristic X with a normal distribution, mean μ, and standard deviation σ. When calculating capability indices, it is assumed that the process is stable, the observations are statistically independent, and they follow a normal distribution. The basic capability index C_p is defined as:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma} \tag{2}$$

where USL is the upper specification limit,

LSL is the lower specification limit,

σ is the standard deviation of the characteristic.

A disadvantage of this index is that it does not allow assessment of process centring – the distance of the process mean μ from the midpoint of the specifications. Therefore, it is considered only a measure of potential process capability (Montgomery, 2020, p. 328).

Unlike the C_p index, the C_{pk} index responds to a shift of the process mean from the midpoint of the specifications. It is calculated as:

$$C_{pk} = min\ (C_{pu},\ C_{pl}), \text{ where } C_{pu} = \frac{\text{USL}-\mu}{3\sigma}\ ;\quad C_{pl} = \frac{\mu-\text{LSL}}{3\sigma} \tag{3}$$

Unlike C_p, which is an indicator of potential process capability, C_{pk} is an indicator of actual process capability because it responds to the deviation of the process mean μ from the midpoint of the specifications in the case of a two-sided interval. The parameters μ and σ are usually unknown and need to be estimated by point estimates $\hat{\mu}$ and $\hat{\sigma}$. The estimated values $\hat{C}_p$ and $\hat{C}_{pk}$ are then obtained by replacing μ and σ in equations (2) and (3) with their point estimates $\hat{\mu}$ and $\hat{\sigma}$ (Terek & Hrnčiarová, 2004; Benková, Bednárová, & Bogdanovská, 2024; Montgomery, 2020; Tošenovský & Tošenovský, 2019).

Let the characteristic X have a normal distribution with a mean of μ_0 at the midpoint of the specifications (the process is ideally centred) and a standard deviation of σ. Then the upper and lower specification limits, USL and LSL, are:

$$\text{USL} = \mu_0 + z\sigma \tag{4}$$

$$\text{LSL} = \mu_0 - z\sigma \tag{5}$$

where z is the value of the random variable Z with a standard normal distribution and determines the distance of USL and LSL from the midpoint of the specifications μ_0, measured in standard deviations σ. The value of z also determines the sigma level (z-score). A corresponding DPMO value is associated with each z value. When calculating it, a shift of the process mean from the midpoint of the specifications by 1.5σ to the right or left is assumed.

There is a relationship between the z-score and the C_{pk} index. Let us first consider ideal centring of the characteristic at the midpoint of the specifications (the so-called short-term view). Then, C_{pu} and C_{pl} represent the distance of USL and LSL from the midpoint of the specifications, measured in standard deviations σ, divided by three, so: $C_{pk} = C_{pu} = C_{pl} = z/3$. For an ideally centred process, $C_{pk} = C_p$. When considering a shift of the characteristic mean by 1.5 standard deviations to either the right or left (long-term view), then $C_{pk} = (z - 1.5)/3$.

The Sigma to DPMO, Yield to C_{pk} tables, and the Six Sigma Conversion Table show the corresponding values of DPMO, z-score, and C_{pk} for a short-term view in the rows. Chen, Ouyang, Hsu, & Wu (2009) explain the relationship between the z-score and C_{pk}, considering both scenarios: when the process is centred within the specification range and when the process mean shifts by 1.5σ to the right or left.

To estimate the z-score, an approach in which C_{pk} is calculated for a characteristic based on relations (3), and then the z-score is computed using the formula $z = 3C_{pk} + 1.5$, can be used. This method can be applied when estimating the z-score for individual characteristics only, but not when estimating the z-score for the entire process. Another limitation of this approach is that capability indices are not calculated for nominal characteristics.

3 METHODOLOGY

Only continuous and nominal critical-to-quality characteristics (CTQCs) will be considered in a process. We assume that the continuous ones follow a normal distribution with a mean μ and standard deviation σ, while the nominal ones can take on two values: non-defective and defective. Nonconformity is defined as non-fulfilment of a requirement, and defect is defined as non-fulfilment of a requirement related to an intended or specified use (ISO 3534-2, 2014). In this article, each nonconformity – i.e., non-compliance of the value of a critical-to-quality characteristic with the requirements (specified specification limits or performance standards) – is considered a defect[1]. A defect opportunity is any measurable event creating a possible defect (ISO 13053-2, 2011). We assume that each critical-to-quality characteristic is associated with only one type of defect opportunity.

We consider n_{CTQC} critical-to-quality characteristics. The production of a single unit provides n_{CTQC} distinct defect opportunities. The i-th critical-to-quality characteristic creates the defect opportunity of type i. One defect opportunity of type i can result in only one defect of type i on a single unit.

When implementing statistical process control of the characteristic, and when calculating the process capability index C_{pk}, it is essential to estimate the mean and standard deviation of the continuous characteristic when the process is stable. Using them, we estimate the z-score for each continuous critical-to-quality characteristic. The z-score for each nominal critical-to-quality characteristic is

[1] We will assume that any non-compliance of the value of a critical-to-quality characteristic with the requirements will affect the functionality or safety of the product.

estimated from the sample proportion of defective units. Finally, the z-score for the overall process is estimated.

4 PROCEDURE FOR ESTIMATING SIGMA LEVEL FOR CTQCs AND THE ENTIRE PROCESS

Let us first consider only one critical-to-quality characteristic. The DPU in the population is the probability that a unit is defective (the proportion of defective units, the number of defective units per unit). The DPO in the population is the probability that a defect opportunity generates a defect (the proportion of opportunities generating a defect, the number of opportunities that generate a defect per opportunity). When we consider only one critical-to-quality characteristic (continuous or nominal), each unit provides one defect opportunity, and $\mathrm{DPU} = \mathrm{DPO}$.

Suppose that the mean of a continuous characteristic X with a normal distribution is shifted from the midpoint of the specifications μ_0 by k standard deviations σ:

$$\mu = \mu_0 + k\sigma$$

where k is the distance of μ from μ_0 measured in standard deviations σ. We calculate the probability that the unit is non-defective:

$$P(\mathrm{LSL} \leq X \leq \mathrm{USL})$$

Next, we will standardise USL and LSL:

$$z_{\mathrm{USL}} = \frac{\mu_0 + z\sigma - (\mu_0 + k\sigma)}{\sigma} = z - k$$

$$z_{\mathrm{LSL}} = \frac{\mu_0 - z\sigma - (\mu_0 + k\sigma)}{\sigma} = -(z + k)$$

where z_{USL}, z_{LSL} are the values of the random variable Z with the standard normal distribution. Apparently:

$$P(\mathrm{LSL} \leq X \leq \mathrm{USL}) = P(z_{\mathrm{LSL}} \leq Z \leq z_{\mathrm{USL}}) = \Phi(z - k) - 1 + \Phi(z + k)$$

where Φ is the distribution function of the standard normal distribution. The probability that a unit is defective is:

$$\begin{aligned} \mathrm{DPU} &= P(X \leq \mathrm{LSL}) + P(X \geq \mathrm{USL}) = 1 - P(\mathrm{LSL} \leq X \leq \mathrm{USL}) = \\ &= 2 - \Phi(z - k) - \Phi(z + k) \end{aligned} \quad (6)$$

Since we consider only one characteristic, $\mathrm{DPU} = \mathrm{DPO}$, when the number of defects per opportunity is multiplied by 1,000,000, we obtain the number of defects per million opportunities (DPMO).

In the Six Sigma concept's design, it is assumed that the process mean can shift from the target value by 1.5 standard deviations, either to the right or left. This model serves as a standard for comparing process performance based on z-score across various critical-to-quality characteristics. ISO 13053-1(2011) includes Table A.1 — Sigma scores, which presents calculated DPMO values for process sigma levels from 0 to 6, in increments of 0.01. These values are based on the probability of a unit being defective in relation to the upper specification limit only[2] (for further details, see Terek, 2023):

$$\mathrm{DPO} = P(X \geq \mathrm{USL}) = P\left(Z \geq \frac{\mu_0 + z\sigma - (\mu_0 + 1{,}5\sigma)}{\sigma}\right) = 1 - P(Z \leq (z - 1.5)) = \\ = 1 - \Phi(z - 1.5)$$

We will assume that if the critical-to-quality characteristic is continuous, it has a normal distribution, and we know its estimated mean and estimated standard deviation. If it is a nominal one, we assume that we know its estimated proportion of defective units.

First, we will consider each critical-to-quality characteristic separately. For each of them, we will find the sigma level. We will proceed as follows:

1. If the critical-to-quality characteristic is continuous, we will proceed to step 2; if it is nominal, we will proceed to step 3.
2. We will calculate the distance of each of the specification limits from the midpoint of the specifications, measured in standard deviations, and the deviation of the mean of the characteristic from the midpoint of the specifications, measured in standard deviations. We will substitute the calculated values in relation (6) for z and k and calculate the estimated DPO. We will proceed to step 4.
3. We will estimate the DPO value by the estimated value of the proportion of defective units and proceed to step 4.
4. From the estimated DPO, we will calculate the estimated DPMO value and determine the z-score from it.

Finally, we will estimate the sigma level of the entire process by using the estimated DPOs of each critical-to-quality characteristic.

Consider $n_{\mathrm{CTQC}} > 1$ critical-to-quality characteristics. Let DPO_i be the probability that a defect opportunity of type i results in a defect of type i. The types of opportunity are equally numerous, because each type occurs on all units. Then $\mathrm{DPO}_i / n_{\mathrm{CTQC}}$ is the probability that a defect opportunity of any type results in a defect of type i.

The events "a defect of type i will occur" for $i = 1, 2, ..., n_{\mathrm{CTQC}}$ are disjoint, because we are considering opportunities, not units. One defect opportunity of type i can result in only one defect of type i. Then we can calculate the probability that a

[2] The results in Table A1 differ only slightly from the results obtained by calculating the probability that a unit is defective according to equation (6), for $k = 1.5$.

defect opportunity of any type results in a defect of any type simply as a sum of probabilities, as follows:

$$\mathrm{DPO} = \sum_{i=1}^{n_{\mathrm{CTQC}}} \mathrm{DPO}_i / n_{\mathrm{CTQC}} \quad (7)$$

After multiplying DPO by 1,000,000, we get DPMO as the number of defects per million opportunities for the entire process, from which the *z*-score of the entire process is determined. The procedure will be demonstrated using an example from ceramic capacitor production.

Example. Ceramic capacitors are being manufactured, and we consider three critical-to-quality characteristics:

X_1 represents capacitance (μF), which indicates the ability to store electric charge. It is a continuous variable with specification limits: LSL = 9.5 μF and USL = 10.5 μF. The midpoint of the specifications is μ_0 = 10 μF. In the process capability analysis, it was determined that the distribution of the variable X_1 is approximately normal. The estimated mean is $\hat{\mu}_1$ = 10.2 μF, with an estimated standard deviation of $\hat{\sigma}_1$ = 0.1 μF. The mean is shifted 0.2 μF to the right of the midpoint of the specifications.

When substituting the values 9.5 and 10.5 for LSL and USL, respectively, $\hat{\mu}_1$ = 10.2 for μ and $\hat{\sigma}_1$ = 0.1 for σ into relations (2) and (3), we obtain:

$$\hat{C}_p = 1.667;\ \hat{C}_{pk} = 1$$

X_2 – equivalent series resistance (Ω) – a continuous variable with specification limits LSL = 0.08 Ω, USL = 0.12 Ω, with the midpoint of the specifications μ_0 = 0.1 Ω. The distribution of the variable X_2 is approximately normal; the estimated mean is $\hat{\mu}_2$ = 0.095 Ω, and the estimated standard deviation is $\hat{\sigma}_2$ = 0.01 Ω. The mean of the variable shifts 0.005 Ω to the left of the midpoint of the specifications.

When substituting the values 0.08 and 0.12 for LSL and USL, respectively, $\hat{\mu}_2$ = = 0.095 for μ and $\hat{\sigma}_2$ = 0.01 for σ into relations (2) and (3), we obtain:

$$\hat{C}_p = 0.667;\ \hat{C}_{pk} = 0.5$$

X_3 – dimensions (width × height × length) (mm) – 3.2 ± 0.2 mm × 1.6 ± 0.2 mm × 1.5 ± 0.1 mm. The dimensions are critical for mounting the ceramic capacitor on a printed circuit board. Compliance with the dimensional specification – a nominal variable with values "non-defective" when all dimensions are within specifications, and "defective" when at least one dimension is out of specification. From a random sample, the sample proportion of defective units was calculated as $\hat{p} = 0.023$.

We will determine the *z*-score for each of the three characteristics and for the overall ceramic capacitor manufacturing process. In Six Sigma practice, results are usually rounded to four or five decimal places. However, intermediate results used in other formulas, such as the calculation of the sigma score, are not rounded.

1. We will determine the *z*-score for X_1

First, we will calculate z_1 and k_1, where z_1 is the distance of each specification limit X_1 from the midpoint of the specifications, measured in standard deviations $\hat{\sigma}_1$, and k_1 is the distance $\hat{\mu}_1$ from μ_0, measured in standard deviations $\hat{\sigma}_1$.

$$z_1 = \frac{\mu_0 - \mathrm{LSL}}{\hat{\sigma}_1} = \frac{10 - 9.5}{0.1} = 5$$

$$k_1 = \frac{\hat{\mu}_1 - \mu_0}{\hat{\sigma}_1} = \frac{10.2 - 10}{0.1} = 2$$

After substituting z_1 and k_1 for z and k in (6), we get:

$\widehat{\mathrm{DPO}}_1 = 2 - \Phi(z_1 - k_1) - \Phi(z_1 + k_1) = 2 - \Phi(5 - 2) - \Phi(5 + 2) = 2 -$
$- \Phi(3) - \Phi(7) = 2 - 0.998650102 - 1 = 0.001349898$

of which:

$\widehat{\mathrm{DPMO}}_1 = \widehat{\mathrm{DPO}}_1 \cdot 1{,}000{,}000 \approx 1\ 350.$

In ISO 13053-1 (2011), on page 28, there is Table A.1 – Sigma scores. Based on a DPMO value of 1350, the *z*-score is approximately 4.5 (according to Table A.1). There is also an alternative method to determine the *z*-score. We know that:

$$1 - \widehat{\mathrm{DPO}}_1 = \Phi(z - 1.5)$$

After substituting 0.001349898 for $\widehat{\mathrm{DPO}}_1$ in the last relation, we get:

$$0.998650102 = \Phi(z - 1.5) = \Phi(z_{0.998650102})$$

Using the NORM.S.INV function in Excel, we find $z_{0.998650102} \approx 3$. Then $(z - 1.5) \approx 3$, which gives us $z \approx 4.5$. The sigma score for the process with respect to ceramic capacitor capacitance is approximately 4.5.

2. Let's calculate the *z*-score for X_2

First, we calculate z_2 and k_2, where z_2 is the distance of each of the specification limits of X_2 from the midpoint of the specifications, measured in standard deviations $\hat{\sigma}_2$, and k_2 is the distance of $\hat{\mu}_2$ from μ_0, measured in standard deviations $\hat{\sigma}_2$.

$$z_2 = \frac{\mu_0 - \mathrm{LSL}}{\hat{\sigma}_2} = \frac{0.1 - 0.08}{0.01} = 2$$

$$k_2 = \frac{\hat{\mu}_2 - \mu_0}{\hat{\sigma}_2} = \frac{0.095 - 0.1}{0.01} = -0.5$$

After substituting z_2 and k_2 for z and k in (6), we get:

$\widehat{\mathrm{DPO}}_2 = 2 - \Phi(z_2 - k_2) - \Phi(z_2 + k_2) = 2 - \Phi(2 + 0.5) - \Phi(2 - 0.5) = 2 -$
$- \Phi(2.5) - \Phi(1.5) = 2 - 0.993790335 - 0.933192799 = 0.073016866$

$\widehat{\mathrm{DPMO}}_2 = \widehat{\mathrm{DPO}}_2 \cdot 1{,}000{,}000 \approx 73{,}017$

When finding the z-score, we put:

$$1 - \widehat{\mathrm{DPO}}_2 = \Phi(z - 1.5)$$

After substituting 0.073016866 for $\widehat{\mathrm{DPO}}_2$ in the last relation, we get:

$$0.926983134 = \Phi(z - 1.5) = \Phi(z_{0.926983134})$$

Using the NORM.S.INV function in Excel, we find $z_{0.926983134} \approx 1.45$. Then $(z - 1.5) \approx 1.45$, which gives us $z \approx 2.95$. The same result you can also find at ISO 13053-1 (2011), Table A.1 – Sigma scores for $\widehat{\mathrm{DPMO}}_2 = 73{,}017$. The sigma score of the process with respect to the equivalent series resistance is approximately 2.95.

3. We calculate the z-score for X_3

The third characteristic is the conformity to the dimensional specification – a nominal variable with values of “non-defective” when all dimensions are within specification, “defective” when at least one dimension is not within specification.

For nominal variables, capability indices are not calculated. The sample proportion of defective units, $\hat{p} = 0.023$, was calculated from the random sample. We can write:

$\widehat{\mathrm{DPO}}_3 = \hat{p} = 0.023$ and $\widehat{\mathrm{DPMO}}_3 = 23{,}000$

When finding the z-score, we put:

$$1 - \widehat{\mathrm{DPO}}_3 = \Phi(z - 1.5)$$

After substituting 0.023 for $\widehat{\mathrm{DPO}}_3$ in the last relation, we get:

$$0.977 = \Phi(z - 1.5) = \Phi(z_{0.977})$$

Using the NORM.S.INV function in Excel, we find $z_{0.977} \approx 1.995$. Then $(z - 1.5) \approx 1.995$, which gives us $z \approx 3.5$.

The sigma score of the process with respect to the conformity to the dimensional specification is approximately 3.5. The same result you can also find at ISO 13053-1 (2011), Table A.1 – Sigma scores for $\widehat{\mathrm{DPMO}}_3 = 23{,}000$.

We will now calculate the estimated sigma level of the entire ceramic capacitor production process, with three critical-to-quality characteristics. We will calculate according to relation (7):

$$\widehat{\mathrm{DPO}} = \sum_{i=1}^{n_{\mathrm{CTQC}}} \widehat{\mathrm{DPO}}_i \,/ n_{\mathrm{CTQC}} = \frac{0.001349898 + 0.073016866 + 0.023}{3} = \\ = 0.032455588$$

Of which:

$$1 - \widehat{\mathrm{DPO}} = 0.967544412$$

When finding the z-score, we put:

$$1 - \widehat{\mathrm{DPO}} = \Phi(z - 1.5)$$

After substituting 0.032455588 for $\widehat{\mathrm{DPO}}$ in the last relation, we get:

$$0.967544412 = \Phi(z - 1.5) = \Phi(z_{0.967544412})$$

Using the NORM.S.INV function in Excel, we find $z_{0.967544412} \approx 1.84587$. Then $(z - 1.5) \approx 1.84587$, which gives us $z \approx 3.35$. The same result you can also find at ISO 13053-1 (2011), Table A.1 - Sigma scores for $\widehat{\mathrm{DPMO}} = 32{,}456$.

For illustration, let us assume that we have proceeded traditionally. We have randomly selected, say, 1,000 units and found the number of defects of the first to third type in accordance with the estimated $\widehat{\mathrm{DPO}}_i$ – 1, 73 and 23. After substitution in (1), we get:

$$\widehat{\mathrm{DPMO}} = [(1 + 73 + 23)/(1{,}000 \cdot 3)] \cdot 1{,}000{,}000 \approx 32{,}333$$

In ISO 13053-1 (2011), Table A.1 – Sigma scores, we find that the corresponding $z \approx 3.35$. We obtained the same result as when applying the proposed method.

When determining the z-score based on the estimated DPMO, we use a standard model that assumes a shift of the process mean by 1.5 standard deviations to the right from the midpoint of the specifications and considers only the upper specification limit. This is not inconsistent with the procedure for calculating DPO and DPMO, which considers the actual shift in the mean.

Let us consider X_1. From the point of view of process performance, it is ultimately irrelevant whether the value $\widehat{\mathrm{DPO}}_1$ is achieved with a shift of the mean by two standard deviations to the right, combined with a smaller variance, or whether we consider a theoretical model that assumes a change in the mean by 1.5 standard deviations to the right with a larger variance. Originally, the estimated values were $\hat{\mu}_1$= 10.2 µF and $\hat{\sigma}_1$= 0.1 µF. The estimated mean of the characteristic is shifted from the midpoint of the specifications μ_0 = 10 µF, by two standard deviations to the right. The value $\widehat{\mathrm{DPO}}_1$ = 0.001349898. If we consider the model situation – a shift of the mean by 1.5 standard deviations to the right – then for $\hat{\mu}_1$ = 10.16667 µF and $\hat{\sigma}_1$ = 0.11111 µF (the mean is shifted from 10 to the right by 1.5 · 0.11111= = 0.166665), we obtain, according to relation (6), approximately the same value

$\widehat{\mathrm{DPO}}_1$. With a smaller shift of the mean, we would obtain the same $\widehat{\mathrm{DPO}}_1$ value for a larger standard deviation.

A similar interpretation applies to variable X_2 in the example. Originally, the measured values were $\hat{\mu}_2$= 0.095 Ω and $\hat{\sigma}_2$= 0.01 Ω. The estimated mean of the characteristic is shifted from the midpoint of the specifications μ_0 = 0.1 Ω, by 0.5 standard deviations to the left. The value $\widehat{\mathrm{DPO}}_2$ = 0.073016866. If we consider the model situation – a shift of the mean by 1.5 standard deviations to the left – then for $\hat{\mu}_2$ = 0.089830509 Ω and $\hat{\sigma}_2$ = 0.006779661 Ω (the mean is shifted from 0.1 to the left by 1.5 · 0.006779661 = 0.010169491), we obtain, according to relation (6), approximately the same $\widehat{\mathrm{DPO}}_2$. With a larger shift of the mean, we would obtain the same $\widehat{\mathrm{DPO}}_2$ value for a smaller standard deviation. Variable X_3 is nominal, and the sample proportion value can be used to estimate DPO_3. The theoretical model serves as a standard for comparing the performance of different processes using the z-score.

In the example, the process sigma score with respect to the capacitance of the ceramic capacitor is approximately 4.5, with respect to the equivalent series resistance it is approximately 2.95, and with respect to compliance with the dimensional specification it is approximately 3.5. The sigma score of the entire ceramic capacitor production process is approximately 3.35. This indicates that the “weakest point” of the ceramic capacitor manufacturing process is compliance with the specification limits for the equivalent series resistance. The proposed method thus enables improving priority setting and planning of Six Sigma projects.

5 CONCLUSIONS

The article aimed to propose a procedure for using parameter estimates of the distribution of critical-to-quality characteristics obtained from statistical process control or process capability analysis, when estimating the process sigma level for individual critical-to-quality characteristics, as well as for the entire product production process. The proposed procedure allows this estimation to be performed using estimates of the means and standard deviations of continuous characteristics and estimates of proportions of defective units of nominal characteristics.

The given method effectively estimates the z-score for both individual characteristics and the overall process, accounting for both continuous and nominal critical-to-quality characteristics. The benefit of this procedure is that another random sample is not needed to estimate the DPMO. Another advantage of the proposed procedure is that the DPO, DPMO, and z-score estimates can be obtained for each characteristic immediately after estimating the respective means, standard deviations, and proportions. Then, a simple method enables us to determine the DPO, DPMO, and z-score estimates for the entire production process. This is important information for decision-making whether to develop a Six Sigma project for the product production process and, if so, which characteristics have a low *z*-score and should be addressed as a priority in the project.

When implementing statistical process control or determining process capability, the means and standard deviations for continuous characteristics and the proportions of defective units for nominal critical-to-quality characteristics are estimated. It would be a shame not to immediately use this information to estimate their *z*-score, because these estimates can be very helpful in finding processes suitable for Six Sigma projects and in structuring them.

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